IMBSA Tutorial

Finite Degradation Structures

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Finite Degradation Structures

1. Mathematical Framework

Finite Degradation Structures **(FDS)** are the most general mathematical framework of **combinatorial reliability/safety models** such as fault trees, reliability block diagrams, etc.

Combinatorial models: describe the state of the **system** as a **combination** of the states of its **components** or **subsystems.**

System.state = Function(C1.state, C2.state, ..., Cn.state)

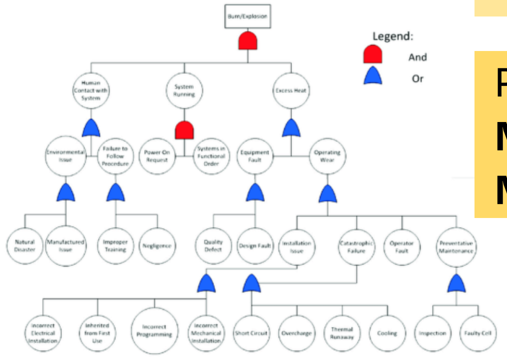
States of system and components can be seen as variables taking their values into finite domains.
The type of the function depends on the domain of variables.

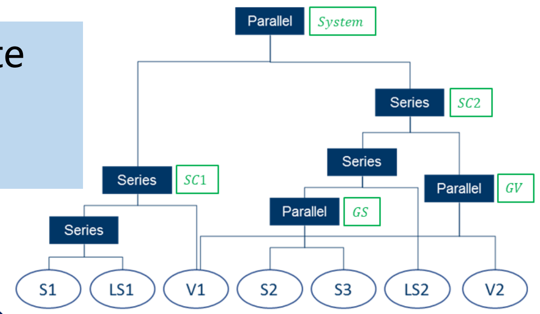
Finite Degradation Structures

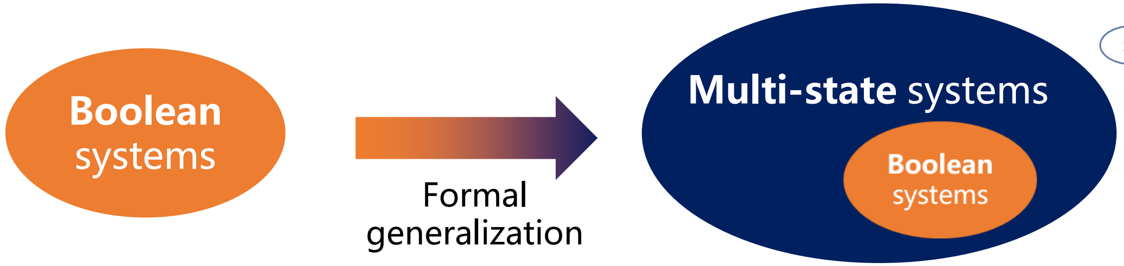
1. Mathematical Framework

More precisely, **FDS** extend the fault tree analysis (FTA) from Boolean systems into **multi-state systems**. FDS generalize formally almost all the notions used in FTA, including:

FTA		FDS
Basic events	Extended →	State variables
Top/intermediate events	→	Flow variables
Logic gates	→	Multi-valued operators
Modeling		
Probability of top event	Extended →	Probability of each state
Minimal path sets	→	**Maximal scenarios**
Minimal cutsets	→	**Minimal scenarios**
Assessment		







Finite Degradation Structures

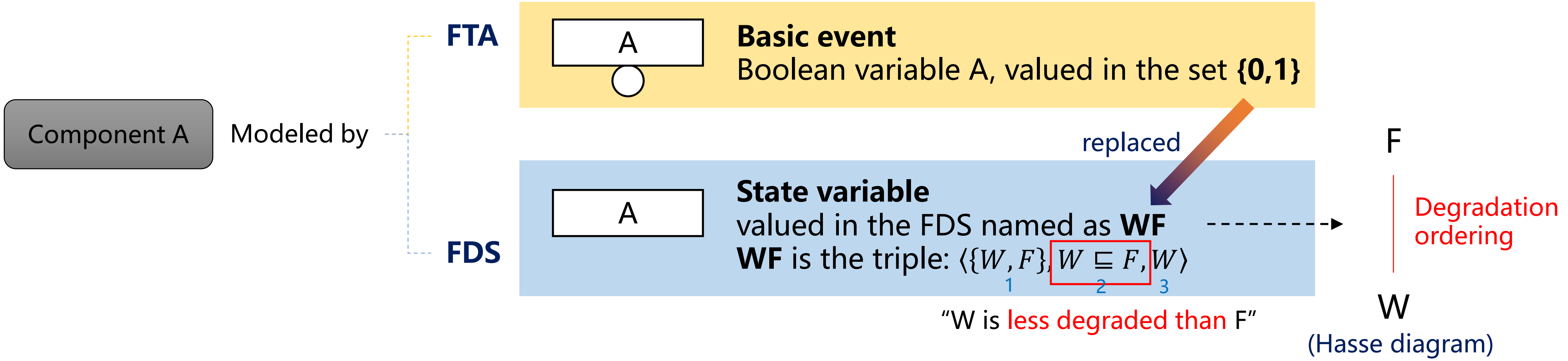
1. Mathematical Framework

FDS are mathematically meet-semi-lattices equipped with probability measure.

A meet-semi-lattice is a triple $\langle \Theta, \sqsubseteq, \bot \rangle$ *s.t.*

- Θ is a **finite set** → 1 State space
- $\sqsubseteq$ is a **partial order** defined over Θ → 2 Degradation order among states
- $\bot$ is the **least element** in Θ with respect to $\sqsubseteq$ → 3 Least degraded state

Example. Boolean component, can be either working (W) or failed (F)



Finite Degradation Structures

2. Modeling Components

Example. Three-state component

	FTA	FDS	
Component A Working (W) Degraded (D) Failed (F)	A=W, A=D, A=F	A valued in the FDS named **WDF**	F D W
	There is no difference between component A and B, except the name of states.	There is a **structural difference** between A and B. This difference impacts on the performance of the system made by A and B.	
Component B Working (W) Failed to open (Fo) Failed to close (Fc)	B=W, B=Fo, B=Fc	B valued in the FDS named **WFoFc**	Fo Fc W

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2. Modeling Components

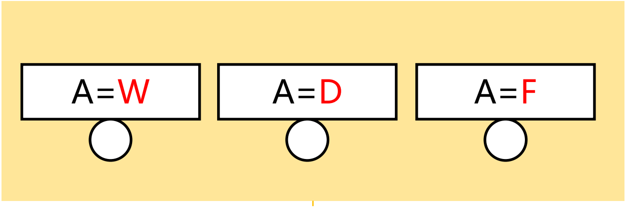
Example. Three-state component

FTA

FDS

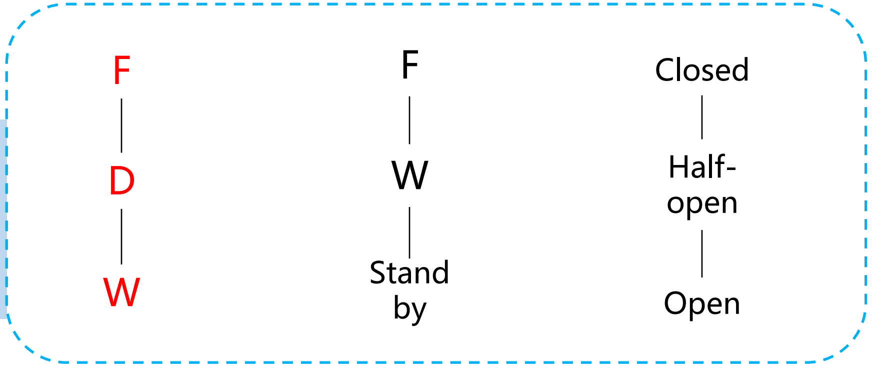
Component A

Working (W)
Degraded (D)
Failed (F)



A

valued in the FDS named **WDF**

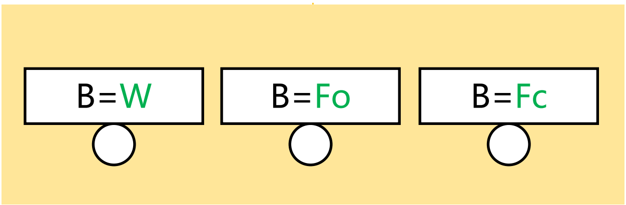


There is no difference between component A and B, except the name of states.

Similar situations
Similar structures
Similar performances?

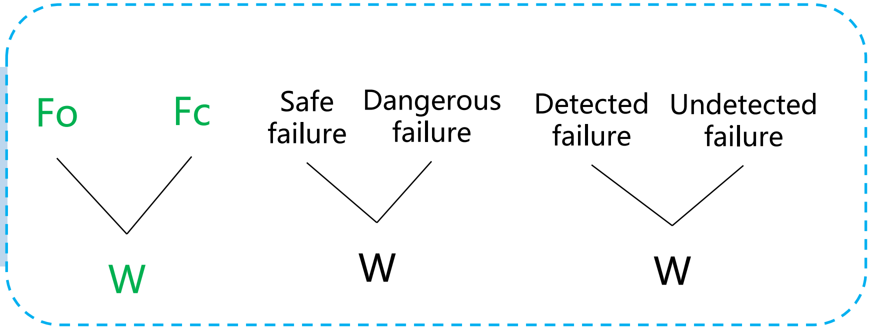
Component B

Working (W)
Failed to open (Fo)
Failed to close (Fc)



B

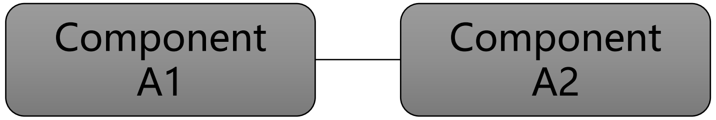
valued in the FDS named **WFoFc**



Finite Degradation Structures

3. Modeling Systems

Example. Series composition of three-state components



Working (W)
Degraded (D)
Failed (F)

Working (W)
Degraded (D)
Failed (F)

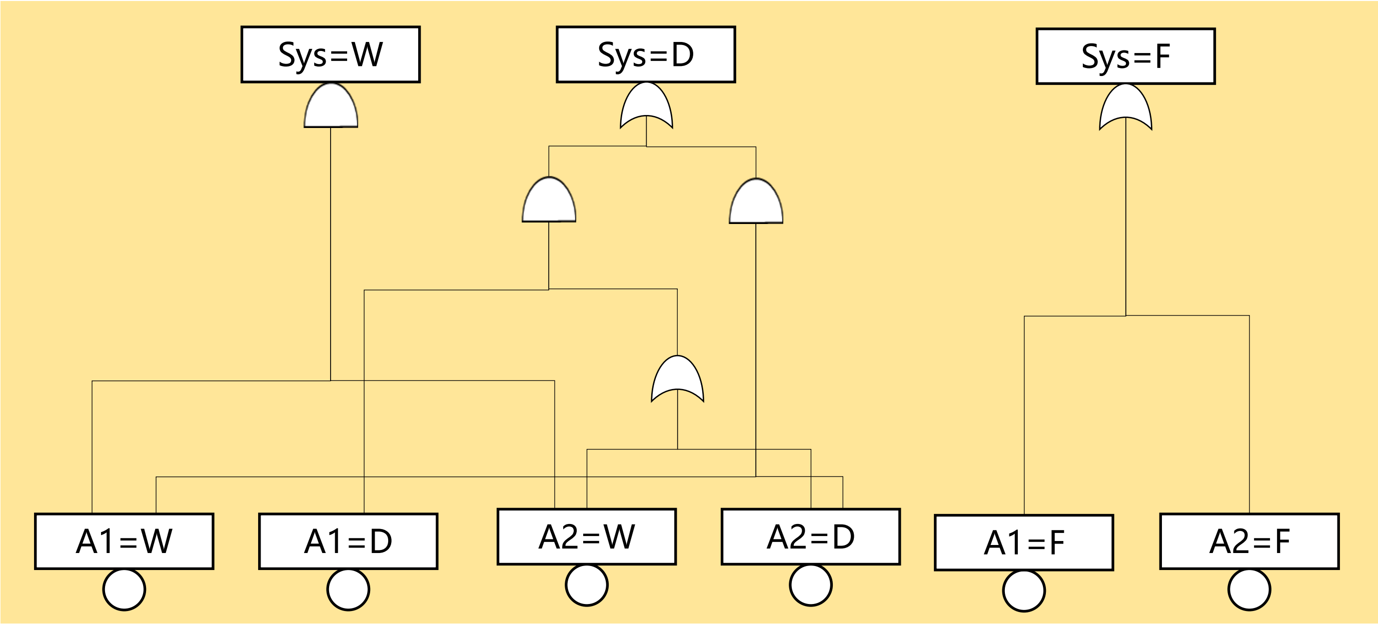
Series operator

A2

	v	W	D	F
	W	W	D	F
A1	D	D	D	F
	F	F	F	F

Valuation table

FTA

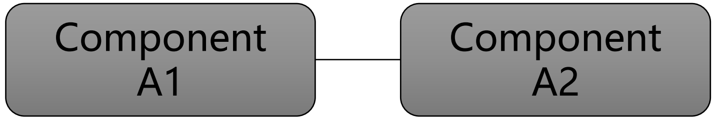


(Sys=W) := WW
(Sys=D) := WD + DW + DD
(Sys=F) := WF +DF + FF + FD + FW

Finite Degradation Structures

3. Modeling Systems

Example. Series composition of three-state components



Component A1	Component A2
Working (W)	Working (W)
Degraded (D)	Degraded (D)
Failed (F)	Failed (F)

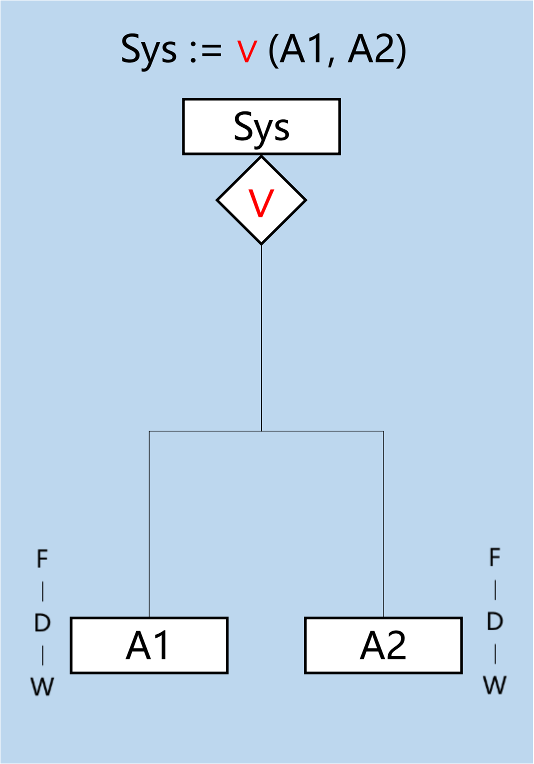
Series operator

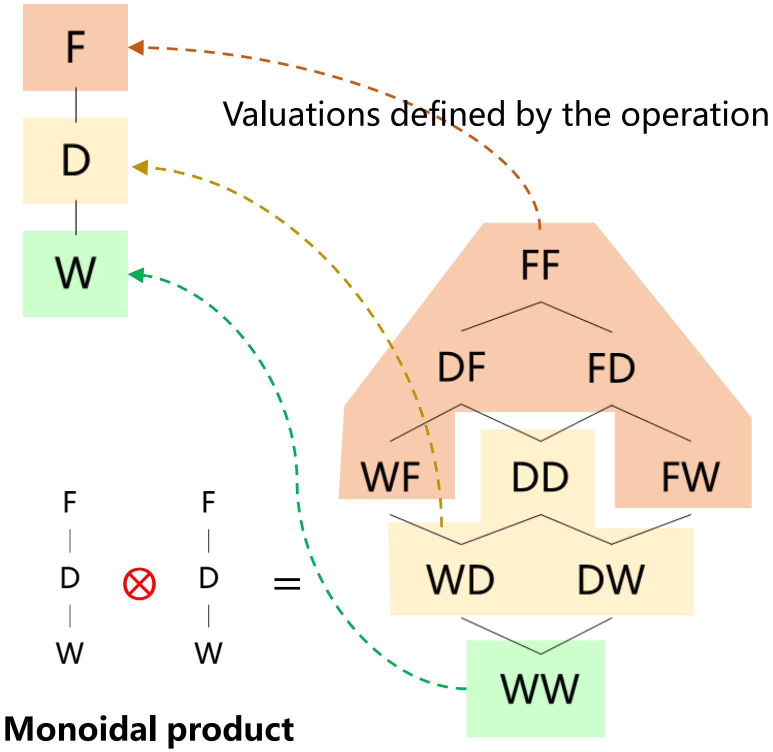
A2

A1	∨	W	D	F
	W	W	D	F
	D	D	D	F
	F	F	F	F

Valuation table

FDS





Monoidal product on FDSs

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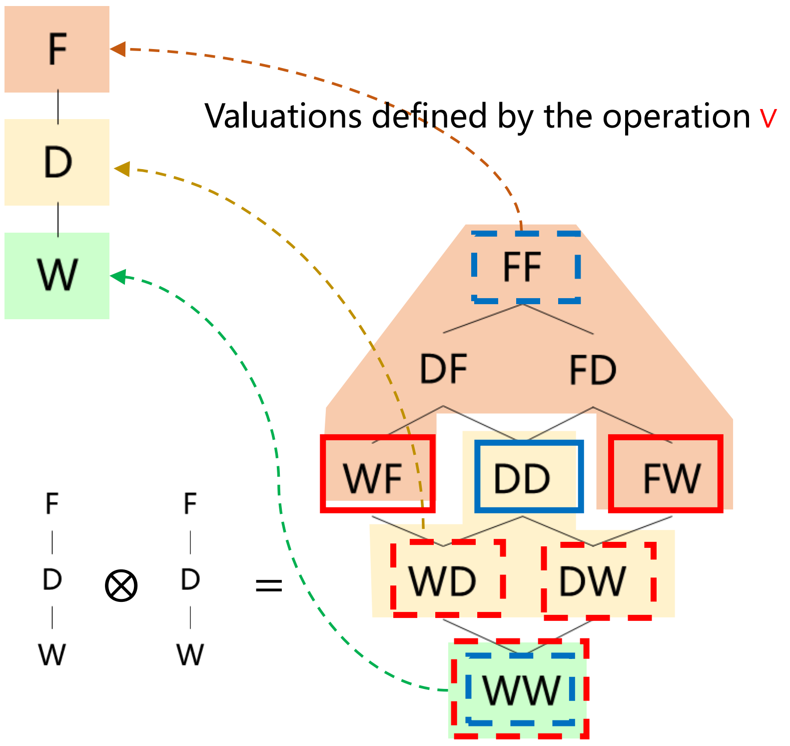
4. Assessing Models

Most highlighted contribution of FDS --- Critical scenarios for multi-state systems

- Minimal scenarios : least degraded state(s) that the system enters into an undesired state ~ minimal cutsets (Extended from)
- Maximal scenarios : most degraded state(s) that the system still remains in an optimal state ~ minimal path sets



Maximal scenarios — Least upper bounds

Minimal scenarios — Greatest lower bounds

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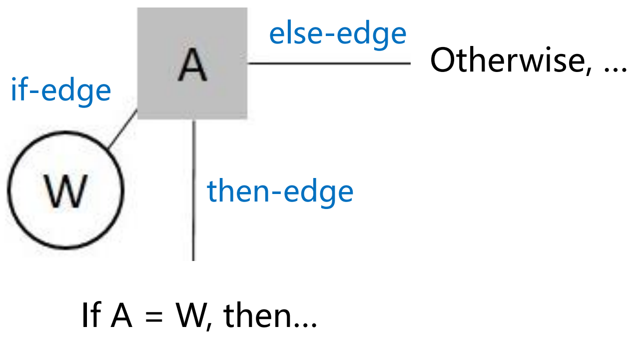
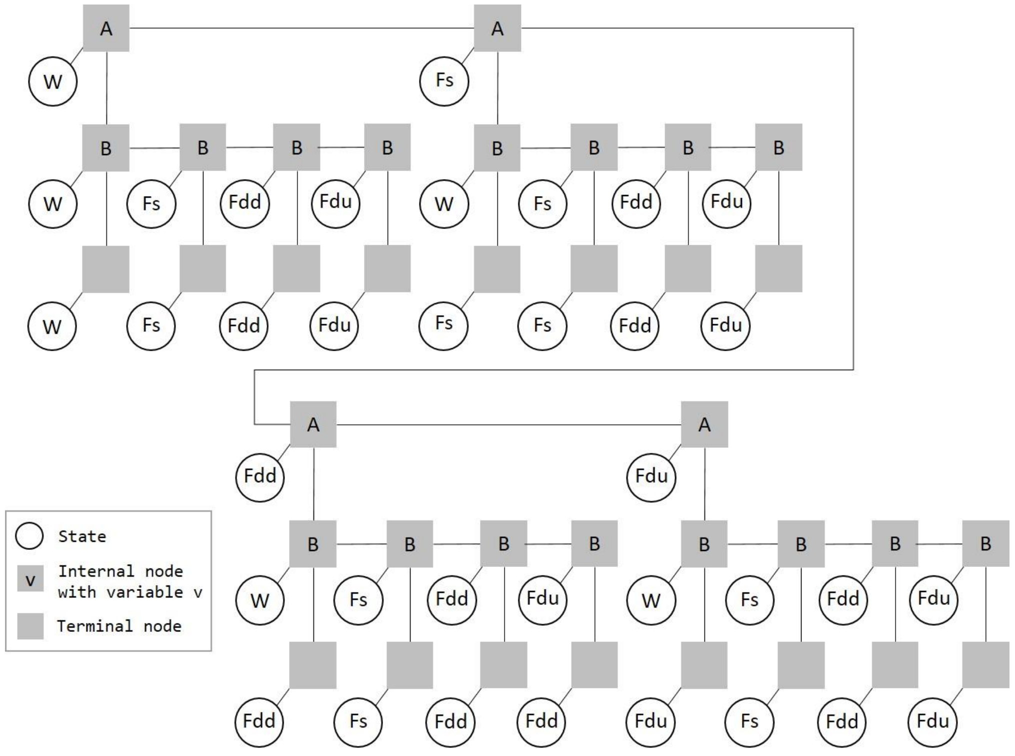
4. Assessing Models

Extended decision diagram

for assessing multi-state models built on FDSs

- **Terminal nodes**: valuation results from different paths
- **Internal nodes**: labeled with variables





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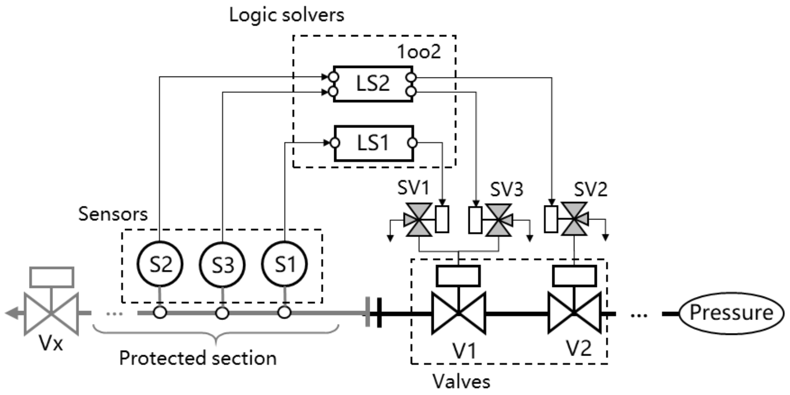
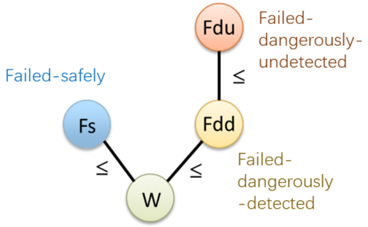
5. Case study

Example. HIPPS (High Integrity Pressure Protection Systems)



Step 1. **Components**: S1, S2, S3, LS1, LS2, V1, V2

whose valuation domains are customized FDSs



Step 2. **Operators**

Series	W	Fs	Fdd	Fdu
W	W	Fs	Fdd	Fdu
Fs	Fs	Fs	Fdd	Fdd
Fdd	Fdd	Fs	Fdd	Fdd
Fdu	Fdu	Fs	Fdd	Fdu

Parallel	W	Fs	Fdd	Fdu
W	W	Fs	W	W
Fs	Fs	Fs	Fs	Fs
Fdd	W	Fs	Fdd	Fdu
Fdu	W	Fs	Fdu	Fdu

Step 3. **Formulate** the **model** of the system

```
SafetyChannel1 := Series(Series(S1,LS1),V1)
SafetyChannel2 := Series(Series(SensorGroup,LS2),ValveGroup)
SensorGroup := Parallel(S2,S3)
ValveGroup := Parallel(V1,V2)
System := Parallel(Safetychannel1,SafetyChannel2)
```

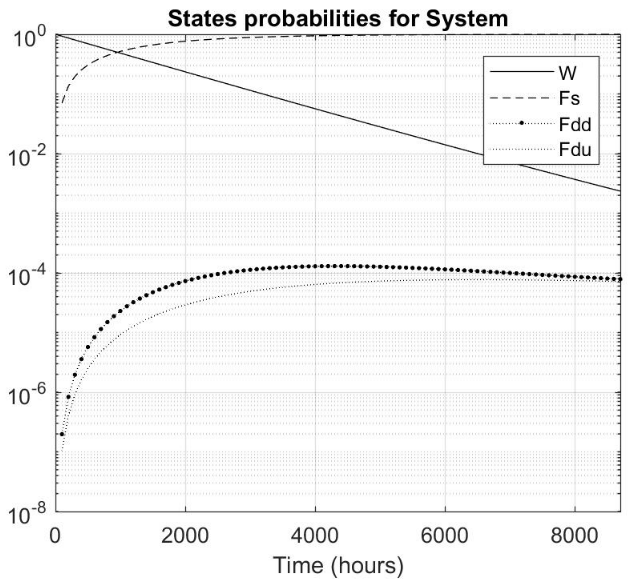
Finite Degradation Structures

5. Case study

Example. HIPPS (High Integrity Pressure Protection Systems)

Step 4.
Assessment

1. Probabilistic results



2. Critical scenarios

(made up of **state combinations** of the 7 components)

$$\{(W,W,W,F_{du},F_s,F_{dd},F_{du}),\\ (W,W,W,F_s,F_s,F_{dd},F_{du}),\\ (W,W,W,F_s,F_{du},F_{dd},F_{du}),\\ (W,W,W,F_{du},F_{du},F_{dd},F_{du}),\\ (F_{du},F_{dd},F_{du},W,F_{du},W,W),\\ (F_{du},F_{dd},F_{du},F_{du},W,W,W),\\ (F_{du},F_{dd},W,W,F_{du},W,F_{du}),\\ (F_{du},F_{dd},W,F_{du},W,W,F_{du}),\\ (F_s,F_{dd},W,W,F_{du},W,F_{du}),\ldots\}$$

Maximal scenarios that the system is still in the working state **W**

16/170/4^7

$$\{(W,W,F_{du},W,W,W,F_{du}),\\ (W,W,F_{du},W,W,F_{dd},W),\\ (W,W,F_{du},F_{dd},F_{dd},W,W),\\ (W,F_{dd},W,F_{dd},F_{du},W,W),\\ (W,F_{dd},W,F_{du},F_{dd},W,W),\\ (F_{dd},W,W,F_{dd},F_{du},W,W),\\ (F_{dd},W,W,F_{du},F_{dd},W,W),\\ (F_{du},W,W,W,W,F_{dd},W),\\ (F_{du},W,W,F_{dd},F_{dd},W,W)\}$$

Minimal scenarios that the system is failed into **Fdu** state

9/316/4^7

of **Critical**/mapped/total scenarios

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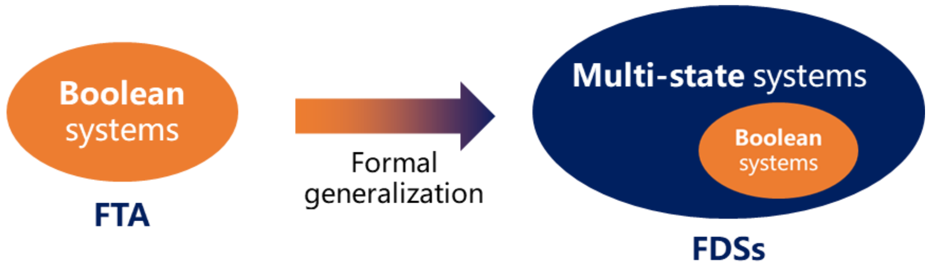
6. Conclusion

- FDS **unify** Boolean and multi-state combinatorial models into one framework, from both theoretical and practical point of view.



- In particular, FDS make it possible to generalize (and to revisit) the central notion of minimal cutsets.

- FDS also provide **interfaces** with systems architectural decompositions (to synchronize with the system design), …

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7. References

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Thanks for your listening.

Questions?