An engineering approach to optimize system design or spare parts inventory

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Abstract

The aim of this paper is to show how the stochastic Petri nets, commonly used in reliability field to model the functional and dysfunctional behaviour of industrial systems and to assess their dependability, are also able to give some interesting information on their global performance, which can be exploited from a technical and economical point of view. By this way the Petri nets can be used, in some cases, to identify the best configuration of system being under design and to determine the right number of spares to be kept in store. Thus, this engineering approach could be an alternative to optimization methods.

Keywords : Petri nets, Monte-Carlo simulation, design optimization spare parts allocation

1. Introduction

Conflicting objectives of safe operation and economic service appear when engineers design an industrial plant or system. To perform their work they have to carry out a twofold approach to overcome both reliability, availability and safety constraints and economic ones. In their task, the design engineers are confronted with several choices to create a system which satisfies the above constraints. These choices concern both the type and the number of components making up this system and their assembly configuration.

The same kind of conflict exists when one must determine how many spares have to be kept in store to ensure the good functioning of a system during its life cycle. Marseguerra and his co-authors have clearly described the multiform nature of this conflict and they have proposed a unique approach, combining the use of genetic algorithms and Monte-Carlo simulation, to find the best configuration for systems under design ([1]) or the right number of spare parts required for the different components of a system ([2]), contrary to many other methods ([3],[4]) already addressing these problems, which are not relevant when applied to realistic size systems. In this paper we also propose a unique engineering approach to treat these two kinds of optimization problems. It is based on the use of Petri nets (PN) model and is divided into three parts as follows :

- The first one is devoted to the modelling of the different design options as a whole, thanks to the great flexibility of the PN-model.

- In the second step, we animate these PN-models by means of a Monte-Carlo (MC) simulation technique, which determines the firing delay of each transition of these models. This way, we discriminate a reduced set of best options among all the possible ones.
- The third and last part is dedicated to the final choice among the options previously selected. This search for the optimum is driven by an objective function defined in ([1]).

The main interest of our approach consists in its simplicity. The PN-modelling is a concise way to model the behaviour of a great variety of configurations of a given system, and the MC-simulation applied to these different models gives explicit indicators enabling a good selection of the best options.

The remainder of this paper is organized as follows. In the next section we remind the safety and economic constraints and the cost function considered by Marseguerra et al. in their papers and used in this one. Section 3 is devoted to the description of the systems chosen as illustrative examples. In section 4 we develop our approach applied to the first system. Some related PN models are presented and numerical results are given. Section 5 summarizes the problem of the optimal allocation of spare parts and our solution. A short conclusion ends this paper.

2. Definition of the net profit function (npf)

2.1. NPF for optimal system configuration design

The considered problem has been very clearly presented in ([1]). We have only to remind that, in order to guide the selection of the possible configurations, the design engineer must define an objective function, which accounts for all the relevant aspects of system operation. The net profit drawn by the system during its mission time T_M is considered as the right objective function. This net profit objective function G (G stands for gain) can be written as follows (see ([1]) :

$$G = P - (C_A + C_R + C_D + C_{ACC}) \quad (1)$$

where : $P = P_t \int_0^{T_M} A(t)dt$ is the system profit in which P_t is the amount of money per unit time paid by the customer for the system service, supposed constant in time, and A(t) is the system availability at time t ; $C_A = \sum_{i=1}^{N} C_i$ is the acquisition and installation cost of all N components of the system ; $C_R = \sum_{i=1}^{N} C_{Ri} \int_0^{T_M} I_{Ri}(t)dt$ is the repair cost of all N components of the system with C_{Ri} being the cost per unit time of repair of component i, and $I_{Ri}(t)$ being a characteristic function equal to 1 if the component i is under repair at time t, 0 otherwise ; $C_D = C_U \int_0^{T_M} [1 - A(t)]dt$ is the amount of money to be paid to the customer because of missed delivery of agreed service during downtime, with C_U (constant in time) being the monetary penalty per unit of downtime.

$C_{ACC} = \sum_{i=1}^{N_{ACC}} I_{ACC,i} \cdot C_{ACC,i}$ is the amount of money to be paid for damages and consequences to the external environment in case of an accident. N_{ACC} is the number of different types of accidents that can occur to the system, $C_{ACC,i}$ is the accident premium to be paid when an accident of type i occurs, $I_{ACC,i}$ is an indicator variable equal to 1 if an accident of type i has happened, 0 otherwise. We assume that after an accident the system cannot be repaired and must be shut down.

2.2. NPF for optimal allocation of spare parts

In this case, the Net Profit Function (NPF) is simpler. It is given in ([2]) as follows :

$$G = P - (C_A + C_S + C_D) \qquad (2)$$

where P, C_A and C_D are defined in the previous sub-section 2.1, and C_S is the cost associated to the handling of the N_{SP} spare units held in store, S_i being the cost of managing spare units i, i.e : $C_S = \sum_{i=1}^{N_{SP}} S_i$

3. Description of the systems chosen to illustrate our approach

3. 1. case 1 : Optimization of the system design

This system has been proposed in ([1]). It consists of N = 3 nodes (sub-systems) in series. For each node there are four possible options (see table 1), so that there are 64 potential configurations at the system level : a_i-b_j-c_k, with i, j, k = 1 to 4. Moreover the following assumptions are made :all components a of node A are equal, all standby are cold, all components are repairable, and have exponentially distributed failure and repair times, the number of available repairmen is always sufficient.
Tables 2 and 3 contain respectively the system and components data.

Table 1. Potential node configurations

Node	Configuration 1	Configuration 2	Configuration 3	Configuration 4
A	1-out-of-1 G	1-out-of-2 G	1-out-of-3 G	2-out-of-3 G
B	1-out-of-1 G	1-out-of-1 G + 1 standby	1-out-of-1 G + 2 standby	1-out-of-1 G + 3 standby
C	1-out-of-1 G	1-out-of-1 G + 1 standby	1-out-of-1 G + 2 standby	1-out-of-1 G + 3 standby

Table 2. System data

Profit per unit time P_t (10^3€ year^{-1})	0.94
Downtime penalty per unit time C_u (10^3€ year^{-1})	3.00
Accident reimbursement cost C_{ACC} (10^3€)	420
Mission time T_M (year)	30

Table 3. Component failure, repair rates and purchase costs

Component	Failure rate λ_i (year^{-1})	Repair rate μ_i (year^{-1})	Purchase cost C_i (10^3€)	Repair cost C_{Ri} (10^3€ year^{-1})
a	2.6×10^{-3}	1.0×10^{-1}	0.7	2.5
b1	5.3×10^{-3}	3.0×10^{-1}	0.3	1.5
b2	3.6×10^{-3}	1.0×10^{-1}	0.3	0.5
b3	4.7×10^{-3}	3.0×10^{-1}	0.7	4.0
b4	2.6×10^{-3}	1.0×10^{-1}	0.7	2.5
c1	8.1×10^{-3}	5.0×10^{-1}	4.0	21.0
c2	5.3×10^{-3}	3.0×10^{-1}	6.0	29.0
c3	7.0×10^{-3}	5.0×10^{-1}	2.0	12.0
c4	4.2×10^{-3}	3.0×10^{-1}	8.0	48.5

3. 2. case 2 : Optimal allocation of spare parts

The system here considered is made up of $N_c = 4$ components in series requiring different kinds of spares. The following assumptions are made : all components have exponentially distributed failure times, they are not repairable but their replace time is negligible.

Because the possible number of spare components for each node equals to 15, there are 16 potential configurations for each node and 65536 (16^4) possible spare parts allocations for the system. Tables 4 and 5 contain respectively the component characteristics and the system cost data (in arbitrary units).

Table 4. Failure rates purchase costs, spare management costs for components

Type	Failure rate λ_i (h^{-1})	Component cost C_i	Spare management cost S_i
1	$5\ 10^{-4}$	500	100
2	$1\ 10^{-3}$	500	100
3	$5\ 10^{-3}$	500	100
4	$1\ 10^{-2}$	500	100

Table 5. System cost data

Profit per unit time P_t	50
Downtime penalty per unit time C_U	5
Mission time T_M	1000

4. PN-approach applied to the first system

As mentioned in section 1, the first step of our approach concerns the modelling of the behaviour of each possible configuration of each node. Thus, we have respectively built 6, 4 and 4 Petri nets for nodes A, B and C. All these PN are very small (3 places and 6 transitions in the worst case). Figure 1 shows the PN-models related to node C.

The key point of our method lies in its second step which is devoted to the discrimination of a few configurations among all possible ones. This discrimination is made by examining the numerical results obtained by MC-simulation applied on the previous PN-models and is based only on simple considerations concerning the behaviour of each node configuration, as explained in subsection 4.1. This procedure enables us to reduce drastically the number of potential options of each node.

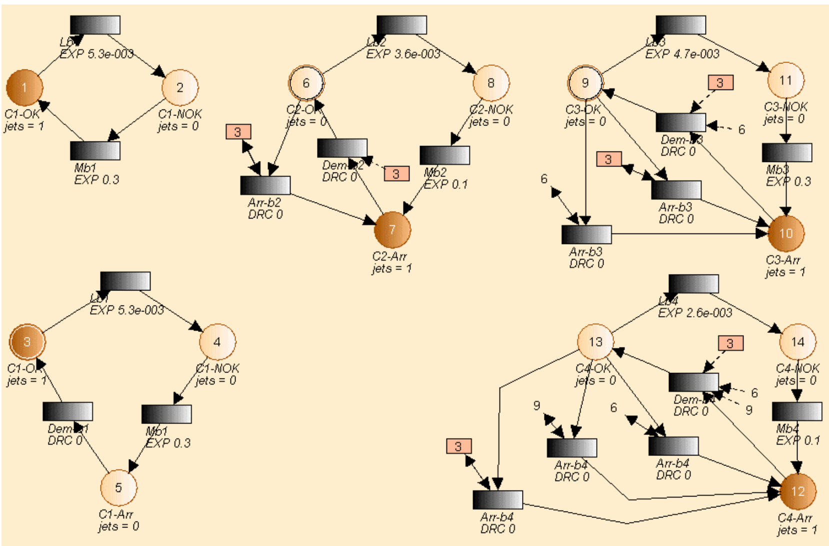

Fig. 1. Petri nets related to node C

4. 1. Reduction of the potential configurations number

- The numerical results concerning the node A are summarized in table 6

Table 6. Results obtained for node A

Performances / Configurations	Average cumulative production time (year)	Average availability	Average cumulative repair time (year)	Acquisition Costs (€)
1	29.475	0.9825	0.525	700
2	30	1	1.05	1400
3	30	1	1.575	2100
4	29.96	0.9987	1.575	2100

Keeping in mind the expression of the net profit function and the fact that node A is composed by a unique type of component, we are able to eliminate the options 3 and 4 by comparing their performances to those of option 2. Thus, only options 1 and 2 are selected at the end of this first step.

. The main numerical results related to node B are in table 7

Table 7. Results obtained for node B

Performances / Configurations	Average cumulative production time (year)	Average availability	Average cumulative repair time (year)	Acquisition Costs (€)
1	29.535	0.9845	0.465	1500
2	29.994	0.9998	0.477	2000
3	29.998	0.9999	0.477	6000
4	29.998	0.9999	0.477	8500

By examinating the above table, one is able to conclude that options 3 and 4 can be eliminated by comparison with option 2. This conclusion is confirmed by the fact that standby components b_3 and b_4 are no really activated during the whole mission time T_M (10^7 Monte-Carlo trials have been performed). This shows the acquisition of these components is not relevant. So, as previously, only configurations 1 and 2 must be retained.

. The case of node C is more complex due to the consequences induced at environmental level by its failure. To take into account this fact, we must also estimate the reliability of node C under its different configurations (see table 7). We have checked the component c_4 has never been on demand during the whole mission time T_M. Then option 4 can be eliminated. Moreover, due to the combination of the important amount of money to be paid in case of an accident and the rather bad reliability of option 1, the latter can also be eliminated. Finally options 2 and 3 are selected.

Table 7. Average availability and reliability of each configuration of node C

Performances / Configurations	Average availability	Reliability
1	0.9851	0.8878
2	0.9999	0.9989
3	1	1
4	1	1

Thus, the reduction of the potential configurations number has been made node by node and without computing and comparing the NPFs of the concerned options.

4. 2 Selection of the best system configuration.

Now, we have to choose the best configuration of the global system among the eight remaining options. This second discrimination is made not on the system as a whole, but successively on each node, as follows :

To discriminate between options 1 and 2 of node A, we compare the net profit functions $G(A_1)$ and $G(A_2)$ of the global system by computing their difference :

$$\Delta G = G(A_1) - G(A_2) \tag{3}$$

Where $G(A_1)$ (resp. $G(A_2)$) concerns the configuration made up of option 1 (resp. 2) of node A, associated with a given combination (the same in $G(A_1)$ and $G(A_2)$) of nodes B and C. The expression (3) becomes :

$$\Delta G = 2012.5 + 3940 \int_0^{T_M} A_B(t) . A_C(t) . [A_{A1}(t) - A_{A2}(t)] dt \tag{4}$$

Because $A_{A1}(t) < A_{A2}(t)$, the second term of the previous sum is negative. To check if the sign of ΔG is positive for all configurations of subsystems C and B, we must compute it with the highest values of $A_B(t)$ and $A_C(t)$, which correspond to the second option of these nodes. Thus, we actually find $\Delta G > 0$. This indicates option 1 of node A is better than option 2. Then it will be retained.

By applying the same procedure to node B, we find option 2 is better than option 1. Option 2 is conserved.

Once again, to discriminate between options 2 and 3 of node C, we determine the sign of the difference $\Delta G = G(C_3) - G(C_2)$. We have :

$$\Delta G = 3940 \int_0^{T_M} A_A(t) . A_B(t) . [A_{C3}(t) - A_{C2}(t)] dt$$

$$- [C_{C3} - C_{C2} + C_{R3} .TR_{C3} - C_{R2..} TR_{C2}) + C_{ACC} (P_{ACC3} - P_{ACC2})] \tag{5}$$

Where Cc_i, C_{Ri}, TR_{Ci} and P_{ACCi} are respectively the acquisition cost, the repair cost, the cumulative repair time and the probability of an accident related to the option C_i, with i = 2,3.

C_{C1}, C_{C2}, C_{Ri} and C_{ACC} are given in tables 2 and 3, T_{RC2} and T_{RC3} are estimated by the cumulative sojourn (or staying) time in the PN-places corresponding respectively to the repair of options 2 and 3 components, and P_{ACCi} corresponds to the unreliability over T_M of the option i. The computing of the expression (5) shows that $\Delta G = G(C_3) - G(C_2)$ is negative. So, we can claim option 2 of node C is better than option 3 and then confirm the choice given in ([1]). The optimal system configuration thus obtained is shown in figure 3.

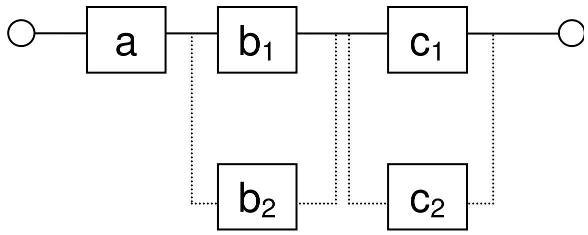

Fig. 3. The optimal system configuration.

5. PN-approach applied to the second system

For the second cas studied in this paper (see its description in paragraph 3.2), the problem of the optimal allocation of spare parts is equivalent to the problem of optimal allocation of cold standby components devoted to each basic component making up the system, because all of them are not repairable and their replace time is negligible.

According to the numerical data given in tables 4 and 5, the expression (2) can be rewritten as.

$$G = P_t \int_0^{T_M} A(t).dt - \left[\sum_{j=1}^{N} Cj + \sum_{j=1}^{N_{SP}} S_j + C_U \,.\int_0^{T_M} [1 - A(t)]\, dt \right]$$

with $N = N_C + N_{SP}$. Then, we have :

$$G = (P_t + C_U) \int_0^{T_M} A(t).dt - \sum_{j=1}^{N_{SP}} (Cj + S_j) - \sum_{j=1}^{N_C} C_j - C_U.T_M \tag{6}$$

Finally (6) can be formulated as follows :

$$G = 55 \int_0^{T_M} A(t).dt - 600\ (k_1 + k_2 + k_3 + k_4) - 7000 \tag{7}$$

where $\int_0^{T_M} A(t).\,dt$ is the mean staying time in the working states of the global system and k_i is the number of spare units of type i.

To obtain the best configuration of the global system in accordance with its gain given by (7), our approach optimizes the number of spares of subsystems by considering them one by one. To do that, a pair comparison between all the sixteen possible options of each subsystem must be performed. Because a such rough procedure would be very tedious, we have carried out a refined comparison procedure as explained here below.

Let us consider two system configurations S_1 and S_2, which differ only from their subsystems of type 1, named respectively A_2 and A_1. From (7) we can deduce configuration S_2 is better than configuration S_1, if the following inequality is satisfied :

$$\Delta G = G(S_2) - G(S_1) > 0 \tag{8}$$

If A_1 and A_2 are two consecutive options of type 1 (i.e A_2 has one spare more than A_1 : k vs k-1), expression (8) can be rewritten as follows :

$$\Delta G = 55 \int_0^{T_M} A_B(t).A_C(t).A_D(t).\left[A_{A2}(t) - A_{A1}(t)\right]dt - 600 > 0 \tag{9}$$

Expression (9) induces the following one :

$$\int_0^{T_M} A_B(t).A_C(t).A_D(t).\left[A_{A2}(t) - A_{A1}(t)\right]dt > \frac{600}{55} = 10.909$$

and finally the above expression implies the last one hereafter :

$$\int_0^{T_M} \left[A_{A2}(t) - A_{A1}(t)\right]dt = \sum_{i=0}^{k} ST_i - \sum_{i=0}^{k-1} ST_i = ST_k > 10.909 \tag{10}$$

where ST_k is the staying time of subsystem A in its working state k, after both its basic component and its first (k-1) spares have failed. In our approach four Petri nets have thus been built. Each of them models all the possible configurations of one subsystem (type i, with i = 1 to 4) as shown in figure 4

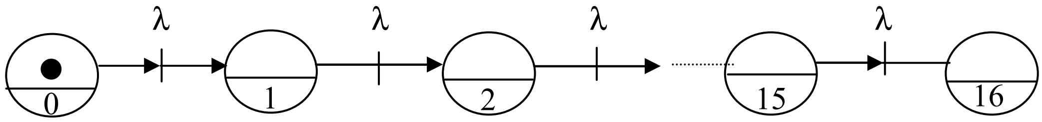

Fig. 4. Simple Petri net related to each subsystem

Then ST_k is estimated by the mean staying time in place k (k = 0 to 15) as shown in table 8.

Table 8. Mean staying times (in hours) in place k (k = 0 to 15)

Places \ Subsystems	SS1 $\lambda_1 = 5.10^{-4}$ h^{-1}	SS2 $\lambda_2 = 1.10^{-3}$ h^{-1}	SS3 $\lambda_3 = 5.10^{-3}$ h^{-1}	SS4 $\lambda_4 = 1.10^{-2}$ h^{-1}
0	786.90	632.10	198.70	100.00
1	180.40	264.30	191.90	99.90
2	**28.79**	80.30	175.10	99.80
3	3.51	**19.00**	147.00	99.00
4	0.34	3.66	111.90	97.00
5		0.60	76.80	93.30
6			47.60	87.00
7			26.70	78.00
8			**13.60**	66.70
9			6.40	54.20
10			2.70	41.70
11				30.30
12				20.80
13				**13.60**
14				8.35
15				4.90

We must note the use of a based MC-simulation Petri net procedure is not the unique way to treat this kind of problem, because the state probabilities and the related mean staying times can be easily computed analytically (Poisson process) or by means of a basic Markovian approach. However, we have carried out a simplified procedure combining PN-MC model and the analytical criterion obtained in (10).

This criterion indicates the option with k spares is better than the option with (k-1) spares, for a given subsystem, if the staying time in place k is greater than 10.9. Then, by examinating the table 8, we conclude immediately the optimal system configuration is the following one : $k_1 = 2$; $k_2 = 3$; $k_3 = 8$; $k_4 = 13$. This confirms the result given in ([2]).

6. Summary

The main interest of the presented approach consists in its simplicity and its flexibility. The PN-modelling is a concise way to model the behaviour of a great variety of configurations and the MC-simulation applied to these different models gives explicit indicators enabling a good selection of the best configurations. Thus, common reliability engineers can easily appropriate it. However this PN-approach is obviously less general than the one proposed in ([1]) and ([2]), and must be further tested on systems with dependent subsystems.

References

[1] Cantoni M., Marseguerra M., Zio E.. *Genetic algorithms and Monte-Carlo simulation for optimal plant design*, Reliability Engineering and System Safety, 68, 29-38 (2000)

[2] Marseguerra M., Podofillini L., Zio E., *Use of genetic algorithms for the optimization of spare parts inventory,* Proceedings of the ESREL'01 Conference, **3**, 1523-1530 (2001)
[3] Messinger M., Shooman M. L., *Techniques for optimal spares allocation – a tutorial review.* IEEE Trans. on Reliability, 19, 156-166 (1970)
[4] Burton B.M., Howard G. T., *Optimal design for system reliability and maintainability,* IEEE Trans. on Reliability, 20, 56-60 (1971)