Contents

New Insight on Measures of Importance of Components and Systems in Fault Tree Analysis

Y. Dutuit
LAP-ADS, Université Bordeaux I,
33405 Talence Cedex FRANCE
dutuit@hse.iuta.u-bordeaux.fr
O. Lemaire
EDF Pôle Industrie, Service EPTN,
69628 Villeurbane Cedex, FRANCE
olivier.lemaire@edf.fr
A. Rauzy
CNRS-LaBRI, Université Bordeaux I,
351, cours de la Libération
F-33405 Talence cedex FRANCE
rauzy@labri.u-bordeaux.fr

Abstract

In this article, we extend the topics of two previous articles which were devoted respectively to the computation of the most frequently used importance factors and to the definition of an additive contribution factor. We revisit all of these factors from the Binary Decision Diagram (BDD) viewpoint. We propose a new computational method to get the Birnbaum importance factor. This method works for non-coherent systems and any macro-event. We discuss the Fussel-Vesely importance factor. Finally, we present a new additive contribution factor.

1. Introduction

To assess the contributions of components to system failure is of a major interest in system design, failure diagnosis and improvement of system reliability. This contribution reveals components that are critical to system performance. In the fault tree framework, this contribution is captured by the so-called importance factors (IF). Many such factors have been proposed in the literature [1]. Most of the developments devoted to IF are based on classical methods to assess the top event probability of a fault tree, i.e. methods that works with minimal cutsets.

PSAM5

However, it is now known these methods are outperformed by the so-called Binary Decision Diagrams (BDD) technique [2]. With BDDs, not only the assessment of the top event probability is more efficient than with conventional approaches (it is linear in the size of the BDD that encodes the structure function), but also the result is exact (for no approximation is performed). It is therefore of a great interest to revisit IF from the BDD viewpoint.

The remainder of this article is organized as follows. In the next section, a BDD based approach to compute both gates gates reliability and joint-reliability importances is presented. The section 3. is devoted to the extension of the Birnbaum IF. The section 4. discusses the Fussel-Vesely IF. In section 5., an additive importance factor is presented.

2. Gates Reliability and Joint-Reliability Importances

The Birnbaum or marginal IF (MIF) plays a central role in the definition of importance measures. It is defined as follows [3].

$$\mathrm{MIF}(S,e) \quad = \quad \frac{\partial p(S)}{\partial p(e)} \tag{1}$$

where S is the structure function under study, e is a basic event and $p(S)$ and $p(e)$ denote the probabilities of occurrence of respectively S and e. It is easy to verify that the following equalities hold.

$$\mathrm{MIF}(S,e) = p(S|e) - p(S|\bar{e}) = \frac{p(S|e) - p(S)}{1 - p(e)}$$

where $p(S|e)$ denotes the probability that S occurs given that e has occurred.

In reference [4], it is shown that all of the most frequently used IF, such as marginal IF (MIF), critical IF (CIF), Risk Achievement Worth (RAW) and Risk Reduction Worth (RRW), can be computed from elementary probabilities related to the fault tree. Namely, probabilities of basic events, top event and gates. These quantities will be used to compute the gate reliability importance (GRI) and gate joint-reliability importance (JRI).

Gate Reliability Importance. The concept of GRI was introduced in reference [5] to extend the notion of MIF to gates (and not only basic events). The reference [6] showed that the definition that was proposed in [5] was incorrect because the involved partial derivative cannot be used in the case where basic events are shared among gates. In this case, the following definition should be used instead.

$$\mathrm{GRI}(S,G) \quad = \quad p(S|G) - p(S|\overline{G}) \tag{2}$$

where G denotes any function (gate) built over the basic event of S. The equality $\mathrm{GRI}(S,e) = \frac{p(S|G)-p(S)}{1-p(G)}$ still holds in that extended case.

Using the BDD approach, a very simple way to assess $p(S|G)$ is to substitute the rate $\frac{p(S.G)}{p(G}$ for it. This formulation is correct for any functions S and G. The method to assess $\mathrm{GRI}(S,G)$ consists of two main steps: first, BDDs that encode functions S, G and $S.G$ are computed. Second, probabilities $p(S)$, $p(G)$ and $p(S.G)$ are assessed from these BDDs. This algorithm is simpler than the one presented in reference [6]. It runs very quickly, even applied to large fault trees, thanks to the efficiency of BDDs.

Joint Reliability Importance The JRI of two components was introduced in the framework of reliability networks [7]. It is defined as the measure of how two components interact to contribute to the system reliability. The notion of JRI was extended on the one hand to deal with statistical dependencies [8] and on the other hand to be used in the framework of fault trees [9]. In this latter case, the JRI of two gates G_i and G_j is defined as follows.

$$\mathrm{JRI}(S,G_i,G_j) \;=\; p(S|G_i,G_j) + p(S|\overline{G_i},\overline{G_j}) - p(S|G_i,\overline{G_j}) - p(S|\overline{G_i},G_j) \quad (3)$$

As previously, a BDD based algorithm can be easily defined to compute such a quantity.

3. Extended Birnbaum IF

If the structure function S is not coherent (non monotone), the MIF related to a basic event cannot be computed by using any of the previous definitions. However, in reference [4] we suggest the use of what we called exclusive cofactor to do so. The exclusive cofactor $S_e^{\#}$ of the function S and a basic event e is defined as follows.

$$S_e^{\#} \;=\; S_e.\overline{S_{\bar{e}}} \quad (4)$$

where S_e denotes the cofactor of S and e, i.e. the function S in which the constant 1 has been substituted for the variable e. The MIF is then (re)defined as follows.

$$\mathrm{MIF}_{\mathrm{NC}}(S,e) \;=\; p(S_e^{\#}) \quad (5)$$

Consider for instance the function $S = a\bar{b}c + ab\bar{c} + \bar{a}bc$. With the conventional definition, we have: $\mathrm{MIF}(S,a) = p(S|a) - p(S|\bar{a}) = p(b\bar{c}) - p(bc)$. Using exclusive cofactor instead, we have $S_a^{\#} = (b\bar{c} + \bar{b}c).\overline{bc} = b\bar{c} + \bar{b}c$. Therefore, $\mathrm{MIF}(S,a) = p(b\bar{c}) - p(bc) \neq p(b\bar{c} + \bar{b}c) = \mathrm{MIF}_{\mathrm{NC}}(S,e)$.

It worth mentioning that the latter formula differs from that given in reference [10]. By examining the expanded expression of S, it appears that $p(S_e^{\#})$ corresponds exactly to the probability of the critical states of the system. Moreover, it is easy to verify that, in the case where S is a monotone function, $\mathrm{MIF(S,e)} = \mathrm{MIF}_{\mathrm{NC}}(S,e)$. A BDD algorithm to compute $S_e^{\#}$ is proposed in reference [4].

4. The Vesely-Fussel IF

The Fussel-Vesely measure represents the fraction of the risk measure to which a considered basic event e contributes. It is defined by means of minimal cutsets C_1, C_2, …, of S:

$$\mathrm{FV}_1(S,e) = \frac{\sum_{C_i \ni e} p(C_i)}{\sum_{C_i} p(C_i)} \tag{6}$$

FV is twofold. On the one hand, it is a ratio. Roughly speaking, it compares the weight of a given number of scenarii (those that contain e) with the weight of all of the scenarii under consideration. On the other hand, the sum of the probability of minimal cutsets is often used as a convenient approximation of the top event probability. From this latter point of view, the above definition can be seen as an approximation of the following ratio, which is easily assessed by means of BDDs.

$$\mathrm{FV}_2(S,e) = \frac{p(S|e)}{p(S)} \tag{7}$$

Still with the notion of scenarii in mind, a third definition can be given for FV, that compares probabilities of disjunctions of scenarii:

$$\mathrm{FV}_3(S,e) = \frac{p(\bigvee_{C_i \ni e} C_i)}{p(\bigvee C_i)} = \frac{p(\bigvee_{C_i \ni e} C_i)}{p(S)} \tag{8}$$

This third quantity can be also assessed by means of BDDs.

It would be interesting to have a complete picture of the cases where definitions 6, 7 and 8 converge or/and diverge and to determine which one (if any) better corresponds to the physical notion FV is supposed to capture.

5. An Additive Importance Factor

The additivity is one the desirable features of a "good" IF, because it makes easy to compare the contributions of the different components to a global quantity. Unfortunately, none of the usual IF exhibits this property. This is the reason why one of us proposed a new IF, AIF(S, G), in [11]. This IF extends the critical IF, CIF(S, e), first proposed in [12].

$$\text{CIF}(S,e) = \frac{p(e)}{p(S)} \times \text{MIF}(S,e) \tag{9}$$

$$\text{AIF}(S,G) = \sum_{e \text{ occurs in } G} \text{CIF}(S,e) \tag{10}$$

Again, both $\text{CIF}(S,e)$ and $\text{AIF}(S,G)$ can be easily assessed by means of BDDs.

6. Conclusion

To illustrate the efficiency of BDDs, let us mention the results obtained on a medium size fault tree of our benchmark. This benchmark is made of industrial examples. It is available on demand to the authors. The fault tree `baobab1` we consider here comes from nuclear industry. It is made of 122 gates and 61 basic events. It has 46188 minimal cutsets whose orders range from 1 to 11. The BDD that encodes `baobab1` is built in 0.02s on a laptop computer. It is made of 7362 nodes. The top event probability is obtained in 0.02s. The MIF for all of the basic events is computed in 0.87s. These running times are very illustrative of the BDDs efficiency.

BDDs are not only efficient, but also they make it possible to compute exactly values that are only approximated with classical fault tree assessment methods. It is therefore of a great interest to revisit the notion of IF in order to separate clearly three aspects that are too often mixed in the literature: first, the physical notion one tries to capture. Second, the mathematical definition of the quantity one tries to assess. Third, the algorithm used to assess this quantity.

This article, following two previous ones [4, 11], is a step in that direction.

References

[1] I.N. Kovalenko, N.Y. Kuznetsov, and Pegg. *Mathematical Theory of Reliability of Time Dependent Systems with Practical Applications.* Wiley Series in Probability and Statistics, 1997.

[2] A. Rauzy. New Algorithms for Fault Trees Analysis. *Reliability Engineering & System Safety*, 05(59):203–211, 1993.

[3] Z.W. Birnbaum. On the importance of different components and a multi-component system. In P.R. Korishnaiah, editor, *Multivariable analysis II.* Academic Press, New York, 1969.

[4] Y. Dutuit and A. Rauzy. New algorithms to compute importance factors CPr, MIF, CIF, DIF, RAW and RRW. In *Proceedings of the European Safety and Reliability Association Conference, ESREL'99*, volume 2, pages 1015–1020. A.A. Balkema, 1999. ISBN 90 5809 111 2.

[5] N.Z. Cho and W.G. Jung. A new method for evaluation of system-level importance measures based on singular value decomposition. *Reliability Engineering and System Safety*, 25:197–205, 1989.

[6] H.S. Lee, C.H. Lie, and J.S. Hong. A Computation Method for Evaluating Importance Measures of Gates in Fault Tree. *IEEE Transactions on Reliability*, 46(3):360–365, 1997.

[7] J.S. Hong and C.H. Lie. Joint reliability importance of two edges in an undirected network. *IEEE Transaction on Reliability*, 42:17–23, 1993.

[8] M.J. Armstrong. Joint reliability importance of components. *IEEE Transactions on Reliability*, 44:408–412, 1995.

[9] J.S. Hong, H.Y. Koo, and C.H. Lie. Computation of joint reliability importance of two gates in a fault tree. *Reliability Engineering and System Safety*, 68(1):1–5, 2000.

[10] P.S. Jackson. On the s-importance of elements and prime implicants in noncoherent systems. *IEEE Transactions on Reliability*, R-32(1):21–25, 1983.

[11] O. Lemaire. Importance and contribution factor for systems. In *Proceedings of the ESREL'99 conference*, pages 1147–1151, München-Garching, Germany, 1999.

[12] H.E. Lambert. Measures of importance of events and cut sets in fault trees. In R.E. Barlow, J.B. Fussel, and N.D. Singpurwalla, editors, *Reliability and Fault Tree Analysis*, pages 77–100. SIAM Press, 1975.