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Importance factors of coherent systems: A review

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Abstract

Importance factors are indicators of the risk significance of the components of a system. They are widely used in probabilistic safety analyses to rank components according to their contribution to the global risk. In this article, we review definitions and interpretations of importance factors in the case the support model is a coherent fault tree, and failures of components are described by basic events of that fault tree. First, we show that each importance factor characterizes the probability of a certain set of minterms. The notion of critical states, that is, minterms in which failing/repairing the component suffices to fail/repair the system, plays a central role in this process. Then, we discuss assessment algorithms for the two main technologies at hand: minimal cutsets and binary decision diagrams. Finally, we draw some practical conclusions from these developments. This article thus contributes to clarify mathematical and algorithmic foundations of importance factors.

Keywords

Fault trees, importance factors, minimal cutsets, binary decision diagrams

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Introduction

One of the activities of risk assessment is expected to be the ranking of the components of the system under study with respect to their risk significance or their safety significance. Importance factors are probabilistic indicators that aim to capture different aspects of this significance and thus to make it possible to rank components in different ways.[1–6] They are primarily defined for the case the support model is a coherent fault tree, and failures of components are represented by basic events of this fault tree. Most of them have been introduced in the 1970s,[7–9] that is, at a time where the predominant, if not the only, technology to assess fault trees consisted in calculating probabilistic measures from minimal cutsets (MCS).[10,11] For this reason, most of importance factors are usually defined and calculated in terms of MCS.

In the 1990s, a new assessment technology for fault trees came into play: Bryant's binary decision diagrams[12,13] (BDDs). To the noticeable exception of large models of the nuclear industry, BDDs have proved to outperform the MCS technology (see, for example, Rauzy[14] for an overview of their use for risk analyses). Among other advantages over the MCS approach, BDDs make it possible to calculate exact values of probabilistic measures. In 2001, the authors published an article in which they proposed BDD-based algorithms to calculate importance factors[15] (a first step into this direction has been done before by Sinnamon and Andrews[16]). To do so, we derived "pure" mathematical definitions from actual definitions of importance factors (in terms of MCS), therefore separating mathematical concepts from calculation means.

In the cited article, we did not draw, however, all the consequences of this new perspective on importance factors, nor we discussed its relationship with the MCS approach. The aim of this article is to fill this hole by reinterpreting importance factors in terms of minterms. Mathematically speaking, minterms are the atoms of the underlying Boolean algebra. They represent global states of the system under study. This new interpretation clarifies dramatically the mathematical and physical meanings of importance factors. Namely, we show that they can be categorized according to the set of

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minterms they characterize, which is necessarily one of the following three sets:

- The set of minterms that fail both the system and the component;
- The set of minterms that fail the system but not the component;
- The set of critical states, that is, of minterms in which failing/repairing the component suffices to fail/repair the system.

Thanks to this clarification, we can study assessment algorithms for importance factors, based on the two technologies at hand, namely, MCS and BDDs, and draw some conclusions regarding their practical use.

This article contributes therefore to clarify mathematical and algorithmic foundations of importance factors. The remainder of this article is organized as follows:

- Section "Basic definitions and properties" introduces basic definitions and properties.
- Section "MCS" recalls definition of MCS.
- Section "Critical states" introduces the notion of critical states.
- Section "Importance factors of coherent systems" presents definitions and interpretations of importance factors.
- Section "Algorithms" recalls algorithmic principles of their calculations.
- Based on the developments of the previous sections, section "Discussion" discusses their use.
- Finally, section "Conclusion" concludes the article.

Basic definitions and properties

Throughout this article, we consider Boolean formulae built over a denumerable set E of variables and the usual connectives "$\cdot$" (and), "$+$" (or) and "$-$" (not). Variables are also called basic events.

We use uppercase letters, E, A, B, C, possibly with subscripts, to denote basic events. We use lowercase letters, s, t, c, possibly with subscripts, to denote Boolean formulae. We denote by var(s) the variables occurring in the formula s.

Let s be a Boolean formula. A variable assignment of s is a function from var(s) into $\{0,1\}$. Variable assignments are lifted up as usual as functions from formulae into $\{0,1\}$ using the truth tables of connectives. A Boolean formula s is interpreted as the Boolean function $[\![s]\!]$, that is, as the function from variable assignments of s into $0,1$, defined as follows: for any variable assignment σ of s, $[\![s]\!](\sigma) = 1$ if $\sigma(s) = 1$ and 0 otherwise.

In this article, we do not need to distinguish between syntax and semantics. Therefore, we shall assimilate the Boolean formula s with its semantics $[\![s]\!]$.

A literal is either a variable E or its negation $\overline{E}$. We use uppercase letters, L, I, J, possibly with subscripts, to denote literals. We denote by $\bar{L}$ the opposite of a literal L, given that $\overline{\bar{L}} \equiv L$. Let $\mathcal{L}$ be a set of literals, we denote by $\bar{\mathcal{L}}$ the set of negations of literals of $\mathcal{L}$, that is, $\{\bar{L}; L \in \mathcal{L}\}$.

A product is a conjunct of literals that does not contain both a variable and its negation. Let s be a Boolean formula. A minterm of s is a product that contains a literal built over each variable of var(s). We use lowercase Greek letters, σ, τ, π, ρ, possibly with subscripts, to denote products and minterms. We denote as Minterms($\boldsymbol{\varepsilon}$) the set of $2^{|\varepsilon|}$ minterms that can be built over a set of basic events $\boldsymbol{\varepsilon}$.

There is a one-to-one correspondence between variable assignments and minterms (and therefore between Boolean functions and sets or disjuncts of minterms): the variable assignment σ one-to-one corresponds with the minterm π such that π contains the positive literal E if $\sigma(E) = 1$ and the negative literal $\bar{E}$ if $\sigma(E) = 0$. It follows that any Boolean formula s is equivalent to the set of minterms π such that $\pi(s) = 1$. Minterms of Minterms($\boldsymbol{\varepsilon}$) are the atoms of the Boolean algebra built over $\boldsymbol{\varepsilon}$.

For the sake of convenience, we shall use a set theory notation, that is, we shall note $\pi \in s$ when $\pi(s) = 1$ and $\pi \notin s$ when $\pi(s) = 0$. Note also that $\pi \notin s$ if and only if $\pi \in \bar{s}$.

Example (minterms). As an illustration, consider the two formulae $s_1 = A \cdot B + A \cdot C + B \cdot C$ and $s_2 = A \cdot B + \bar{A} \cdot C$. The minterms of s_1 and s_2 are as follows

$$s_1 \equiv A \cdot B \cdot C + A \cdot B \cdot \bar{C} + A \cdot \bar{B} \cdot C + \bar{A} \cdot B \cdot C$$
$$s_2 \equiv A \cdot B \cdot C + A \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C + \bar{A} \cdot \bar{B} \cdot C$$

Let s be a Boolean function and $\mathcal{L} = \{L_1, \ldots, L_k\}$ be a set of literals built over a subset of var(s). We denote by $s|\mathcal{L}$ the Boolean function built over var(s)\var($\mathcal{L}$) as follows

$$s|\{L_1, \ldots, L_k\} \stackrel{def}{=} \{\pi | L_1, \ldots, L_k \cdot \pi \in s\}$$

For the sake of the simplicity, we write $s|L_1, \ldots, L_k$ (instead of $s|\{L_1, \ldots, L_k\}$). The notation $s|L$ is intentionally close to the one used for conditional probabilities because it is really what it means: s given L.

Example ($s|L$). Considering the function s_1 defined above, the following equalities hold

$$s_1|A \equiv B \cdot C + B \cdot \bar{C} + \bar{B} \cdot C = B + C$$
$$s_1|\bar{A} \equiv B \cdot C$$
$$s_1|A, B \equiv C$$
$$s_1|A, \bar{B} \equiv C$$

We can now state the Shannon decomposition.

Property 1 ((logical) Shannon decomposition). Let s be a Boolean formula and E a basic event of var(s). Then, the following equivalence holds

$$s \equiv E \cdot s|E + \bar{E} \cdot s|\bar{E}$$

Throughout this article, we shall assume that basic events are independent from a statistical viewpoint.

The above equivalence is translated in terms of probability by the either of the two equalities that will play an important role later.

Property 2 ((probabilistic) Shannon decomposition). *Let s be a Boolean formula and E a basic event of var(s). Then, the following equalities hold*

$$\Pr\{s\} = \Pr\{E\} \cdot \Pr\{s|E\} + 1 - \Pr\{E\} \cdot \Pr\{s|\bar{E}\} \quad (1)$$

$$\Pr\{s\} = \Pr\{E\} \cdot \Pr\{s|E\} - \Pr\{s|\bar{E}\} + \Pr\{s|\bar{E}\} \quad (2)$$

In the framework on safety and reliability analyses, basic events represent general failures of components of the system under study. Therefore, there is an asymmetry between a positive literal that represents the occurrence of a failure and its negation. The former has in general a much lower probability than the latter.

This asymmetry induces a natural partial order among minterms. The minterm σ is smaller than the minterm π, which we denote by $\sigma \sqsubseteq \pi$, if for any basic event E, $E \in \sigma$ implies that $E \in \pi$. In other words, all components failed in σ are failed in π as well.

A Boolean function s is coherent if for any two minterms σ and π, such that $\sigma \sqsubseteq \pi$, $\sigma \in s$ implies that $\pi \in s$. In other words, in a coherent system, the more the components failed, the more likely the system itself failed. Or to put it the other way round, the repair of a component cannot fail the system.

A formula built over only connective "$\cdot$" and " + " is coherent.

Example (coherence). The above function s_1 is coherent while s_2 is not: $\bar{A} \cdot \bar{B} \cdot C \in s_2$, but $A \cdot \bar{B} \cdot C \notin s_2$.

If a function s is not coherent, it is always possible to consider a coherent upper approximation of s. Namely, the coherent envelope of s, denoted as $\lceil s \rceil$, which is the smallest coherent set of minterms that contains s

$$\lceil s \rceil \stackrel{def}{=} \{\pi \in \text{Minterms}(\text{var}(s)); \exists \sigma, \sigma \sqsubseteq \pi \wedge \sigma \in s\}$$

Obviously, for any function s, $s \subseteq \lceil s \rceil$ and s is coherent if and only if $\lceil s \rceil \equiv s$.

Example (coherence). Consider again $s_2 = A \cdot B + \bar{A} \cdot C$

$$\begin{aligned} s_2 &\equiv A \cdot B \cdot C + A \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C + \bar{A} \cdot \bar{B} \cdot C \\ \lceil s_2 \rceil &\equiv A \cdot B \cdot C + A \cdot B \cdot \bar{C} \boxed{+ A \cdot \bar{B} \cdot C} + \bar{A} \cdot B \cdot C + \bar{A} \cdot \bar{B} \cdot C \\ &\equiv A \cdot B + C \end{aligned}$$

The added minterm is boxed.

MCS

In this section, we recall basic definitions and properties of MCS. Our presentation follows Rauzy[17] (see also Rauzy[14]).

Let $\boldsymbol{\varepsilon}$ be a finite set of variables and let π be a positive product built over some of the variables of $\boldsymbol{\varepsilon}$. We denote by $\lfloor \pi \rfloor_{\boldsymbol{\varepsilon}}$ the minterm obtained by completing π with negative literals built over variable of $\boldsymbol{\varepsilon}$ that do not show up in π.

Example ($\lfloor \pi \rfloor$). Consider the positive products $A \cdot B$ and C. Then, we have $\lfloor A \cdot B \rfloor_{\{A,B,C\}} = A \cdot B \cdot \bar{C}$ and $\lfloor C \rfloor_{\{A,B,C\}} = \bar{A} \cdot \bar{B} \cdot C$.

Let s be a Boolean function and π be a positive product built over some of the variables of var(s). Then

- π is a cutset of s if $\lfloor \pi \rfloor_{\text{var}(s)} \in f$;
- π is a MCS of s if it is a cutset of s and there is no cutset ρ of s such that $\rho \quad \pi$.

We denote by MCS(s) the disjunction of MCS of s. This formal definition of MCS, first given in Rauzy,[17] works equally for coherent and noncoherent formulae.

Example (MCS). Consider again our two functions $s_1 = A \cdot B + A \cdot C + B \cdot C$ and $s_2 = A \cdot B + \bar{A} \cdot C$. We have

$$\begin{aligned} \text{MCS}(s_1) &= A \cdot B + A \cdot C + B \cdot C \\ \text{MCS}(s_2) &= A \cdot B + C \end{aligned}$$

The following property, again established in Rauzy,[17] characterizes coherent functions through their MCS.

Property 3 (MCS and coherence). *Let s be a Boolean function, then s is coherent if and only if $MCS(s) \equiv s$. Moreover, in any case, $\lceil s \rceil \equiv MCS(s)$.*

Example (coherent envelope). Consider again the function $s_2 = A \cdot B + \bar{A} \cdot C$

$$\begin{aligned} s_2 &\equiv A \cdot B \cdot C + A \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C + \bar{A} \cdot \bar{B} \cdot C \\ \text{MCS}(s_2) &\equiv A \cdot B + C \equiv \lceil s_2 \rceil \end{aligned}$$

The calculation of the probability of a set (a disjunction) of MCS could be done through the so-called Sylvester–Poincaré development (also called inclusion–exclusion principle)

$$\begin{aligned} p(\pi_1 + \cdots + \pi_n) = & \sum_{1 \leqslant i \leqslant n} \Pr\{\pi_i\} \\ & - \sum_{1 \leqslant i_1 < i_2 \leqslant n} \Pr\{\pi_{1_1} \cdot \pi_{i_2}\} \\ & + \cdots \\ & + (-1)^{k+1} \sum_{1 \leqslant i_1 < \cdots < i_k \leqslant n} \Pr\{\pi_{1_1} \ldots \pi_{i_k}\} \\ & + \cdots \end{aligned} \quad (3)$$

This development contains, however, 2^n terms to calculate, which would lead to an exponential blow-up. The transformation of the set of MCS into a set of

disjoint products (from which the probability can be easily calculated), although sometimes more efficient than the inclusion–exclusion principle, suffers from the same problem (see, for example, Châtelet et al.[18] for a review of algorithms to calculate sum-of-disjoint products). Therefore, approximations are used. The so-called mincut upper bound (MCUB) consists in taking only the first term of the Sylvester–Poincaré development. Namely

$$\mathrm{MCUB}(\pi_1 + \cdots + \pi_n) \stackrel{def}{=} \sum_{1 \leqslant i \leqslant n} \Pr\{\pi_i\}$$

MCUB is a safe approximation in the sense that for any disjunction U of products, $\Pr\{U\} \leqslant \mathrm{MCUB}(U)$. Moreover, for most of the real-life coherent system s, this approximation is very accurate: $\Pr\{\mathrm{MCS}(s)\} \approx \mathrm{MCUB}(\mathrm{MCS}(s))$.

Another approximation is often used, the so-called rare event approximation (REA), which is as follows

$$\mathrm{REA}(\pi_1 + \cdots + \pi_n) \stackrel{def}{=} 1 - \Pi_{1 \leqslant i \leqslant n}(1 - \Pr\{\pi_i\})$$

We will not use the REA in the sequel.

Critical states

The notion of critical state is the core of the theory of importance factors. Although this notion may sound familiar to the reader, to our knowledge, the presentation we give here is original and completes an interpretation sketched by Vaurio.[19]

Let s be the Boolean function and L be the literal built over a basic event of var(s). We define critical and noncritical states of s with respect to L as follows

$$\mathrm{crit}(s, L) \stackrel{def}{=} \{L \cdot \pi \in s; \bar{L} \cdot \pi \in \bar{s}\}$$
$$\overline{\mathrm{crit}}(s, L) \stackrel{def}{=} \{L \cdot \pi \in s; \bar{L} \cdot \pi \in s\}$$

Intuitively, a minterm $L \cdot \pi$ of s is critical with respect to the literal L if by flipping the value of L we change also the value of s.

Example (critical states). Consider again the function $s_1 = A \cdot B + A \cdot C + B \cdot C$. We have

$$\mathrm{crit}(s_1, A) = A \cdot B \cdot \bar{C} + A \cdot \bar{B} \cdot C$$
$$\overline{\mathrm{crit}}(s_1, A) = A \cdot B \cdot C$$
$$\mathrm{crit}(s_1, \bar{A}) = 0$$
$$\overline{\mathrm{crit}}(s_1, \bar{A}) = \bar{A} \cdot B \cdot C$$
$$\mathrm{crit}(\bar{s}_1, A) = 0$$
$$\overline{\mathrm{crit}}(\bar{s}_1, A) = A \cdot \bar{B} \cdot \bar{C}$$
$$\mathrm{crit}(\bar{s}_1, \bar{A}) = \bar{A} \cdot B \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot C$$
$$\overline{\mathrm{crit}}(\bar{s}_1, \bar{A}) = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

As suggested by the above example, the notion of criticality separates the minterm space into eight zones, according to the three binary choices $s/\bar{s}$, $E/\bar{E}$ and Critical/$\overline{\text{Critical}}$. This separation is graphically illustrated in Figure 1.

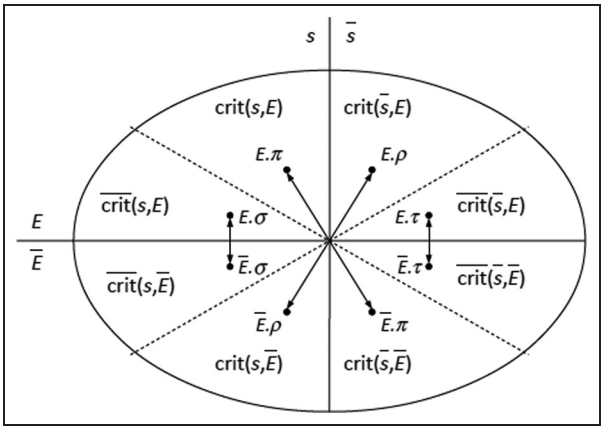

Figure 1. Minterm zones.

This figure illustrates also two simple properties (that follows immediately from the definitions)

$$E \cdot s = \mathrm{crit}(s, E) + \overline{\mathrm{crit}}(s, E)$$
$$\bar{E} \cdot s = \mathrm{crit}(s, \bar{E}) + \overline{\mathrm{crit}}(s, \bar{E})$$

Moreover, zones are paired by flipping the value of E, as established by the following property.

Property 4 (minterm zones). Let s be a Boolean function and E be a basic event of var(s). Then, the following equivalences hold

$$\mathrm{crit}(s, E)|E \equiv \mathrm{crit}(\bar{s}, \bar{E})|\bar{E}$$
$$\mathrm{crit}(s, \bar{E})|\bar{E} \equiv \mathrm{crit}(\bar{s}, E)|E$$
$$\overline{\mathrm{crit}}(s, E)|E \equiv \overline{\mathrm{crit}}(s, \bar{E})|\bar{E}$$
$$\overline{\mathrm{crit}}(\bar{s}, E)|E \equiv \overline{\mathrm{crit}}(\bar{s}, \bar{E})|\bar{E}$$

Intuitively, the above property says that for any minterm $E \cdot \pi$ of $\mathrm{crit}(s, E)$, the minterm $\bar{E} \cdot \pi$ belongs to $\mathrm{crit}(\bar{s}, \bar{E})$ and vice versa. The above logical equivalences translate indeed into probabilities: $\Pr\{\mathrm{crit}(s, E)|E\} = \Pr\{\mathrm{crit}(\bar{s}, \bar{E})|\bar{E}\}$, ... In the case, s represents a coherent system, the repair of a component cannot fail the system. Therefore, the following property holds.

Property 5 (minterm zones of coherent systems). Let s be a coherent Boolean function and E be a basic event of var(s). Then, the following equalities hold

$$\mathrm{crit}(s, \bar{E}) = \mathrm{crit}(\bar{s}, E) = 0$$
$$\overline{\mathrm{crit}}(s, E)|E = \overline{\mathrm{crit}}(s, \bar{E})|\bar{E} = s|\bar{E}$$

We have now all the elements to revisit importance factors.

Importance factors of coherent systems

As mentioned in the "Introduction" section, importance factors have been defined at a time MCS were the predominant, if not the only, technology at hand to

assess fault trees. For this reason, they are usually interpreted in terms of MCS. Moreover, only coherent models were considered (the formal extension of the notion of MCS to noncoherent models came later[17]).

In this section, we shall review the main importance factors. We shall give their definitions both in terms of MCS and in terms of minterms and discuss their meanings. We shall also illustrate their interest and drawbacks by means of small examples.

Marginal importance factor

In many scientific domains, parametric models are defined, and a central question is to measure the sensitivity of the model to variations of its parameters. A way to do so is the so-called marginal gain technique which consists of making each of the parameter vary slightly in turn, and to observe, *mutatis mutandis*, the variations induced on the measure(s) at stake.

This typical local sensitivity technique has been used by Birnbaum[7] to express the importance measure he proposed. The marginal importance factor (MIF), denoted by $\mathrm{MIF}(s, E)$, is defined as follows

$$\mathrm{MIF}(s, E) \stackrel{def}{=} \frac{\partial[\Pr\{s\}]}{\partial[\Pr\{E\}]}$$

The MIF can be assessed by calculating first $\Pr\{s\}$ with the regular value of $\Pr\{E\}$, then with a slightly modified value of $\Pr\{E\}$, for example, $\Pr\{E\} + \epsilon$. But it is generally more efficient and anyway more interesting to apply the second form of the Shannon decomposition (equation (2))

$$\begin{aligned}\mathrm{MIF}(s, E) &\stackrel{def}{=} \frac{\partial[\Pr\{s\}]}{\partial[\Pr\{E\}]} \\ &= \frac{\partial[\Pr\{E\} \cdot \Pr\{s|E\} - \Pr\{s|\bar{E}\} - \Pr\{s|\bar{E}\}]}{\partial[\Pr\{E\}]} \\ &= \Pr\{s|E\} - \Pr\{s|\bar{E}\}\end{aligned} \tag{4}$$

Now look at Figure 1. When s is coherent, according to property 5, $\mathrm{crit}(s, \bar{E}) = 0$. Therefore, according to property 4, $\Pr\{s|\bar{E}\} = \overline{\mathrm{crit}}(s, \bar{E})|\bar{E} = \overline{\mathrm{crit}}(s, E)|E$. Since $s|E = s \cdot E|E = (\mathrm{crit}(s, E) + \overline{\mathrm{crit}}(s, E))|E$, the following property holds.

Property 6 (MIF and critical states). Let s be a coherent fault tree and E be a basic event of s, then the following equalities hold

$$\mathrm{MIF}(s, E) = \Pr\{\mathrm{crit}(s, E)|E\} \tag{5}$$

$$\mathrm{MIF}(s, E) = \Pr\{\mathrm{crit}(s, E)\} + \Pr\{\mathrm{crit}(\bar{s}, \bar{E})\} \tag{6}$$

As we shall see, equations (5) and (6) play a central role in importance factors theory. Now let us come back to equation (4) and approximate them by means of MCS. MCS of s can be split into two subsets: those that contain E and those that do not contain E (nor $\bar{E}$).

Using MCUB approximation, we have

$$\begin{aligned}\Pr\{s|E\} &\approx \sum_{E\cdot\pi\in\mathrm{MCS}(s)} \Pr\{\pi\} + \sum_{\pi\in\mathrm{MCS}(s),\, E\notin\pi} \Pr\{\pi\} \\ \Pr\{s|\bar{E}\} &\approx \sum_{\pi\in\mathrm{MCS}(s),\, E\notin\pi} \Pr\{\pi\}\end{aligned}$$

Therefore, applying equality (4), we have

$$\mathrm{MIF}(s, E) \approx \sum_{E\cdot\pi\in\mathrm{MCS}(s)} \Pr\{\pi\}$$

Critical importance factor

One of the inconveniences of the MIF is that it does not take into account the probability of the basic event. So, if two basic events play similar roles, their ranking according to the MIF will be close, even if their probabilities differ by orders. The critical importance factor (CIF), denoted by $\mathrm{CIF}(s, E)$, is an attempt to correct this drawback. It has been introduced by Lambert.[8] It is defined as follows

$$\mathrm{CIF}(s, E) \stackrel{def}{=} \frac{\Pr\{E\} \times \mathrm{MIF}(s, E)}{\Pr\{s\}}$$

By equality (5), $\Pr\{E\} \times \mathrm{MIF}(s, E) = \Pr\{\mathrm{crit}(s, E)\}$. Moreover, the denominator $\Pr\{s\}$ will be the same for all basic events and therefore play no role in the raking of components of a same model. It should be seen as a way to normalize results over different models. $\mathrm{CIF}(s, E)$ is directly a measure of criticality of the component E.

It can be approximated via MCS using MCUB as follows

$$\mathrm{CIF}(s, E) \approx \frac{\sum_{E\cdot\pi\in\mathrm{MCS}(s)} \Pr\{E \cdot \pi\}}{\sum_{\pi\in\mathrm{MCS}(s)} \Pr\{\pi\}} \tag{7}$$

Diagnostic importance factor

The diagnostic importance factor (DIF) does not attempt to measure the criticality of components but rather to determine which component should be looked at first when the system is failed in order to repair it. This notion is of interest in harsh environments where sending a robot, or even worse a human operator, may present serious difficulties. So, the faster one finds the problem the better.

The DIF, denoted by $\mathrm{DIF}(s, E)$, has been introduced by Vesely and Fussel.[9] It is defined as follows

$$\mathrm{DIF}(s, E) \stackrel{def}{=} \Pr\{E|s\}$$

By conditional probability rule, the following equality holds

$$\mathrm{DIF}(s, E) = \frac{\Pr\{E \cdot s\}}{\Pr\{s\}}$$

In other words, DIF(s, E) is the fraction of the system unavailability (or risk) that involves the component failure. Again $\Pr\{s\}$ should be seen as a normalization factor.

DIF(s, E) can be approximated via MCS using MCUB as follows

$$\Pr\{s|E\} \approx \sum_{E\cdot\pi\in \mathrm{MCS}(s)} \Pr\{\pi\} + \sum_{\pi\in \mathrm{MCS}(s),\, E\notin\pi} \Pr\{\pi\}$$

$$\mathrm{DIF}(s, E) \approx \frac{\Pr\{E\} \times \Pr\{s|E\}}{\sum_{\pi\in \mathrm{MCS}(s)} \Pr\{\pi\}}$$

Vesely–Fussel importance factor

As mentioned above, the DIF is often called Vesely–Fussel importance factor after its authors. In some textbooks, however, it is defined as follows (we denote it VF(s, E) for Vesely–Fussel)

$$\mathrm{VF}(s, E) \stackrel{def}{=} \frac{\Pr\left\{\sum_{E\cdot\pi\in \mathrm{MCS}(s)} E\cdot\pi\right\}}{\Pr\left\{\sum_{\pi\in \mathrm{MCS}(s)} \pi\right\}}$$

At a first glance, such a definition seems very close to those we have already seen. In particular, it looks pretty much the same as the interpretation of CIF(s, E) (and not DIF(s, E)) in terms of MCS (using equation (7)). It turns out, however, that VF(s, E) is equivalent to neither CIF(s, E) nor DIF(s, E) as illustrated by the following example.

Example (VF(s, E)). Let $s = A\cdot B\cdot C + A\cdot B\cdot D + C\cdot D$. We have

$$\mathrm{crit}(s, A) = A\cdot B\cdot C\cdot \bar{D} + A\cdot B\cdot \bar{C}\cdot D$$

$$A\cdot s = A\cdot B\cdot C\cdot D + A\cdot B\cdot C\cdot \bar{D} + A\cdot B\cdot \bar{C}\cdot D + A\cdot \bar{B}\cdot C\cdot D$$

$$\sum_{A\cdot\pi\in \mathrm{MCS}(s)} A\cdot\pi = A\cdot B\cdot C\cdot D + A\cdot B\cdot C\cdot \bar{D} + A\cdot B\cdot \bar{C}\cdot D$$

Therefore, in that case

$$\mathrm{CIF}(s, E) < \mathrm{VF}(s, E) < \mathrm{DIF}(s, E)$$

The point is that *the indicator VF(s, E) has no physical meaning!* Its value may vary from CIF(s, E) to DIF(s, E) depending on s and E. MCS are just a calculation artifact. They cannot be used to describe states of the system!

Risk achievement worth and risk reduction worth

The two other main importance factors, widely used in nuclear probabilistic safety assessment (PSA), are the risk achievement worth (RAW) and risk reduction worth (RRW) (also called risk increase factor and risk decrease factor,[20] respectively). They are defined as follows

$$\mathrm{RAW}(s, E) \stackrel{def}{=} \frac{\Pr\{s|E\}}{\Pr\{s\}}$$

$$\mathrm{RRW}(s, E) \stackrel{def}{=} \frac{\Pr\{s|\bar{E}\}}{\Pr\{s\}}$$

RAW measures the increase in system failure probability assuming the worst case of failing component. It is an indicator of the importance of maintaining the current level of reliability for the component.[4] In Wall and Worledge,[2] it is argued that RAW should be used with care, for it is rather rough.

RRW represents the maximum decrease in the risk that may be expected by increasing the reliability of the component. Therefore, this quantity may be used to select components that are the best candidates for efforts leading to improving system reliability. Note that RRW is sometimes defined as $\Pr\{s\}/\Pr\{s|\bar{E}\}$, that is, the inverse of the above definition (e.g. in RiskSpectrum[20]). Taking one definition or the other does change anything but the presentation of the results.

None of these importance factors take into account the probability of failure of the component and both are normalized with the denominator $\Pr\{s\}$. Using MCUB, they can be approximated via MCS as follows

$$\Pr\{s|E\} \approx \sum_{E\cdot\pi\in MCS(s)} \Pr\{\pi\} + \sum_{\pi\in \mathrm{MCS}(s),\, E\notin\pi} \Pr\{\pi\}$$

$$\mathrm{RAW}(s, E) \approx \frac{\Pr\{s|E\}}{\sum_{\pi\in \mathrm{MCS}(s)} \Pr\{\pi\}}$$

$$\mathrm{RRW}(s, E) \approx \frac{\sum_{\pi\in \mathrm{MCS}(s),\, E\notin\pi} \Pr\{\pi\}}{\sum_{\pi\in \mathrm{MCS}(s)} \Pr\{\pi\}}$$

Algorithms

In this section, we shall discuss calculation algorithms and potential problems. As mentioned previously, there are mainly two technologies to assess reliability indicators from fault trees: MCS and BDDs. We shall therefore examine these two technologies in turn.

BDDs

We assume that the reader has basic knowledge about BDDs. Rauzy[14] provides a rather extensive introduction to the use of BDD for both qualitative and quantitative assessments of fault trees.

Recall that a BDD is a directed acyclic graph whose internal nodes are labeled with basic events and terminal nodes are labeled with Boolean constants (i.e. either 0 or 1). Each internal node has a 0-outedge and a 1-outedge. A total order is chosen over basic events so

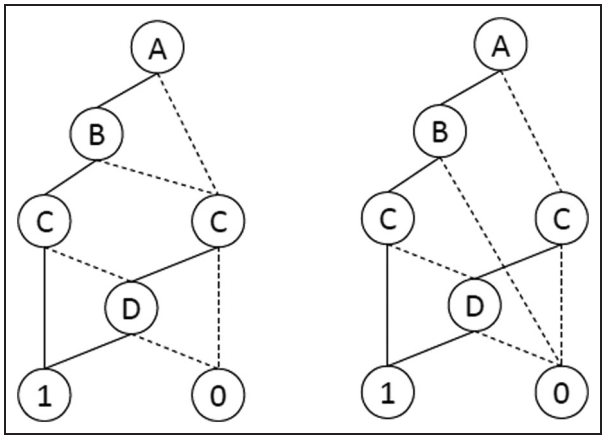

Figure 2. BDD encoding $s = A \cdot B \cdot C + A \cdot B \cdot D + C \cdot D$ (left) and ZBDD encoding *MCS*(*s*) (right).

that a node m labeled with the basic event A is pointing to a node n labeled with the basic event B, then A is smaller (according to the chosen order) than B. Each internal node $n = \langle E, n_1, n_0 \rangle$ encodes the Boolean function $s = E \cdot s_1 + \bar{E} \cdot s_0$, where s_1 and s_0 are the functions encoded by nodes n_1 and n_0 (pointed by 1- and 0-outedges), respectively. The leaves encode the Boolean constants they are labeled with. The BDD encoding the function $s_3 = A \cdot B \cdot C + A \cdot B \cdot D + C \cdot D$ for the lexicographic order is shown in Figure 2 (left).

Thanks to the recursive semantics of BDD, algorithms to calculate probabilistic indicators can be described by means of simple recursive equations. The recursive equations to calculate the $\Pr\{s\}$ are as follows

$$\Pr\{0\} = 0$$
$$\Pr\{1\} = 1$$
$$\Pr\{\langle E, n_1, n_0 \rangle\} = \Pr\{E\} \cdot \Pr\{n_1\} + (1 - \Pr\{E\}) \cdot \Pr\{n_0\}$$

Their correctness follows directly from the Shannon decomposition and the definition of these quantities. There is a subtlety, however, to ensure a linear complexity (in the size of the BDD), results of intermediate computations must be cached: each time the result for a node is to be computed, the algorithm first look up the cache. If the result is already cached, then it is directly returned. Otherwise, it is computed and added to the cache (see Rauzy[14] for more details). The actual algorithm to calculate $\Pr\{s\}$ is shown in Figure 3.

The recursive equations to calculate $\Pr\{s|E\}$ are as follows

$$\Pr\{0|E\} = 0$$
$$\Pr\{1|E\} = 1$$
$$\Pr\{\langle E, n_1, n_0 \rangle|E\} = \Pr\{n_1\}$$
$$\Pr\{\langle F, n_1, n_0 \rangle|E\} = \Pr\{F\} \cdot \Pr\{n_1|E\} + (1 - \Pr\{F\}) \cdot \Pr\{n_0|E\} \quad \text{if } F < E$$
$$\Pr\{\langle F, n_1, n_0 \rangle|E\} = \Pr\{\langle F, n_1, n_0 \rangle\} \quad \text{if } F > E$$

Finally, the recursive equations to calculate $\mathrm{MIF}(s, E)$ are as follows

$$\mathrm{MIF}(0, E) = 0$$
$$\mathrm{MIF}(1, E) = 1$$
$$\mathrm{MIF}(\langle E, n_1, n_0 \rangle, E) = \Pr\{n_1\} - \Pr\{n_0\}$$
$$\mathrm{MIF}(\langle F, n_1, n_0 \rangle, E) = \Pr\{F\} \cdot \mathrm{MIF}(n_1, E) + (1 - \Pr\{F\}) \cdot \mathrm{MIF}(n_0, E) \quad \text{if } F < E$$
$$\mathrm{MIF}(\langle F, n_1, n_0 \rangle, E) = 0 \quad \text{if } F > E$$

It is worth to notice that algorithms to compute $\Pr\{s\}$, $\Pr\{s|E\}$, $\Pr\{s|E\}$ and $\mathrm{MIF}(s, E)$ are linear in the size of the BDD. The other importance factors are computed from these four ones using equalities given in the previous section. Moreover, values of $\Pr\{s\}$, $\Pr\{s|E\}$, $\Pr\{s|E\}$ and $\mathrm{MIF}(s, E)$ can be cached so that all the importance factors of a component can be calculated at once in linear time.

MCS

As shown in the previous section, importance factors can be approximated by means of MCS (using MCUB). There are two main ways to encode a set of MCS: as a sparse matrix or by means of a zero-suppressed binary decision diagram (ZBDD). Sparse matrices (or similar encodings) are rather favored by top-down algorithms,[20,21] while ZBDDs are favored by bottom algorithms.[22]

A sparse matrix for the MCS $A \cdot B + A \cdot C + B \cdot C$ is shown in Figure 4. The idea is to have each MCS encoded as a list of basic events and for each basic event the list of occurrences in the MCS. This double chaining makes it possible to access quickly to MCS containing a basic event.

```
BDDPr(n : BDD)
  if n == 0 then return(0)
  if n == 1 then return(1)
  if cache.HasEntry(n) then return( cache.GetEntry(n) )
  pr = Pr{n.variable}.BDDPr(n.outEdge1) + (1 − Pr{s.variable}).BDDPr(n.outEdge0)
  cache.AddEntry(n, pr)
  return(pr)
```

Figure 3. BDD algorithm to calculate $\Pr\{n\}$.

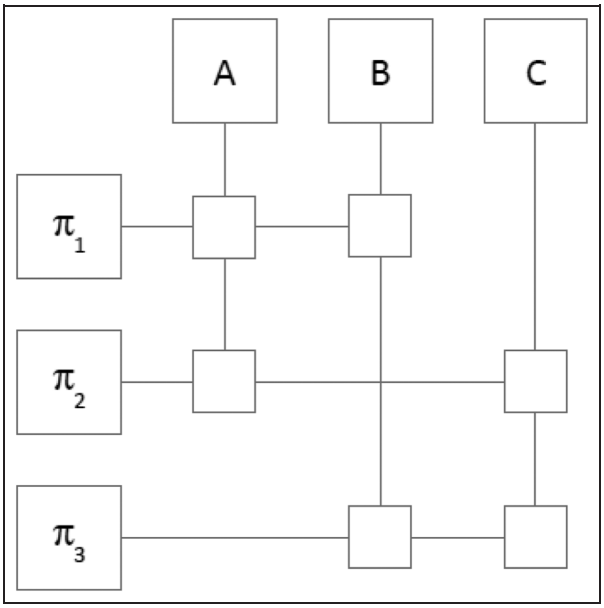

Figure 4. The sparse matrix to encode minimal cutsets $A \cdot B + A \cdot C + B \cdot C$.

```
MCSMIF(s:SparseMatrix, E:BasicEvent)
  mif = 0
  forall c in s.CutsetList(E) do
    pr = 1
    forall F ∈ c do
      if F ≠ E then pr = pr × Pr{F}
    mif = mif + pr
  return(mif)
```

Figure 5. Sparse matrix algorithm to calculate MIF(s, E).

Therefore, the calculations described in the previous section can be implemented efficiently. For instance, the algorithm to compute MIF(s, E) is shown in Figure 5.

Sparse matrices have, however, a drawback: their size is proportional to the number of MCS, or to be more precise, to the number of occurrences of basic events in MCS. Minato's ZBDDs[23] are like regular BDD but with a different interpretation of internal nodes: a node $n = \langle E, n_1, n_0 \rangle$ encodes the disjunction (of MCS) $s = E \cdot s_1 + s_0$, where s_1 and s_0 are the functions encoded by nodes n_1 and n_0, respectively. ZBDD can therefore typically be used to encode sets of MCS. The ZBDD encoding the MCS of the function $s_3 = A \cdot B \cdot C + A \cdot B \cdot D + C \cdot D$ for the lexicographic order is shown in Figure 2 (right).

The algorithms to compute probability and importance factors from ZBDD are indeed quite similar to those for BDD, including for what concerns the caching mechanism. Similarly, they can be described by means of recursive equations. Recursive equations describing the algorithm to compute MCUB approximation of $\Pr\{s\}$ are as follows

$$\begin{aligned}
&\text{MCUB}(0) = 0\\
&\text{MCUB}(1) = 1\\
&\text{MCUB}(\langle E, n_1, n_0 \rangle) = \Pr\{E\} \cdot \text{MCUB}(n_1) + \text{MCUB}(n_0)
\end{aligned}$$

Discussion

Classification

At this point, we can organize importance factors for coherent systems according to the set of minterms (or states) they intend to capture:

- The states in which both the component and the system are failed, as for the DIF and the RAW;
- The states in which the system is failed but the component is working, as for the RRW;
- The critical states, as for the MIF and the CIF.

These sets are illustrated in Figure 6.

There are two other axes to classify importance factors:

Whether they are normalized using $\Pr\{s\}$ as a denominator, as for the CIF, the DIF, the RAW and the RRW;
Whether they take into account the probability of failure of the components, as for the CIF and the DIF.

The overall classification is summarized in Table 1.

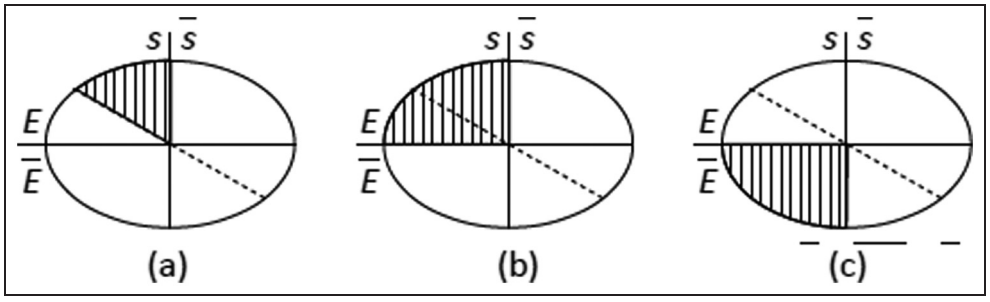

Figure 6. Minterm set described by importance factors (a) crit(s, E), (b) $s \cdot E$, (c) $s \cdot \bar{e} = \overline{\text{crit}}(s, \bar{E})$.

Table 1. Classification of importance factors for coherent systems.

$\Pr\{s\}$	No		Yes	
$\Pr\{E\}$	No	Yes	No	Yes
crit(s, E)	MIF(s, E)	crit(s, E)		CIF(s, E)
$s \cdot E$	$s\|E$	$s \cdot E$	RAW(s, E)	DIF(s, E)
$s \cdot \bar{E}$	$s\|\bar{E}$	$s \cdot \bar{E}$	RRW(s, E)	

Indicators crit(s, E), $s|E$, $s|\bar{E}$, $s \cdot E$ and $s \cdot \bar{E}$ are not used in practice. Note also that Table 1 contains two empty cells. First, there is no normalized MIF. Second, there is no indicator measuring $s \cdot \bar{E}$ and taking into account the probability of the component. In coherent systems, such an indicator would give either $\Pr\{s \cdot \bar{E}\}/\Pr\{s\} = \Pr\{\bar{E}|s\}$ if RRW(s, E) is multiplied by $\Pr\{\bar{E}\}$ or $\Pr\{\overline{\text{crit}}(s, E)\}$ if RRW(s, E) is multiplied by $\Pr\{E\}$.

Eventually, the picture is as follows:

- $s \cdot E$ and DIF(s, E), and to a lesser extent RAW(s, E), measure the proportion of failure states in which the component is failed as well. The higher this proportion, the more the component contributes to the risk. DIF(s, E) is probably the more direct way to assess the risk significance of the component.
- MIF(s, E) measures the probability to be in a "swing state," that is, a state in which flipping the state of the component flips the state of the system. CIF(s, E) measures the proportion of failure states in which repairing the component repairs the system. Note that the proportion of working states in which the failure of the component causes the failure is obtained by the following formula

$$\frac{(1 - \Pr\{E\}) \times \text{MIF}(s, E)}{1 - \Pr\{s\}}$$

- Finally, RRW(s, E) measures the maximum improvement in the reliability of the system one may expect by improving the reliability of the component. To be fully informative, this measure should be considered together with the feasibility and the cost of improving the component. It is not immediately intuitive that RRW(s, E) measures also the proportion of failure states in which the state of the component does not matter. The following equality holds (for coherent systems)

$$\text{RRW}(s, E) = 1 - \text{CIF}(s, E)$$

So, for coherent systems, RRW(s, E) and CIF(s, E) are the two faces of the same medal.

Example (a simple series/parallel system). Let $s = A + B \cdot C$. We have

$$\text{crit}(s, A) = A \cdot B \cdot \bar{C} + A \cdot \overline{B} \cdot C + A \cdot \overline{B} \cdot \bar{C}$$
$$\overline{\text{crit}}(s, A) = A \cdot B \cdot C$$
$$\overline{\text{crit}}(s, \bar{A}) = \bar{A} \cdot B \cdot C$$
$$\text{crit}(s, B) = \bar{A} \cdot B \cdot C$$
$$\overline{\text{crit}}(s, B) = A \cdot B \cdot C + A \cdot B \cdot \bar{C}$$
$$\overline{\text{crit}}(s, \bar{B}) = A \cdot \bar{B} \cdot C + A \cdot \bar{B} \cdot \bar{C}$$
$$\text{crit}(s, C) = \bar{A} \cdot B \cdot C$$
$$\overline{\text{crit}}(s, C) = A \cdot B \cdot C + A \cdot \bar{B} \cdot C$$
$$\overline{\text{crit}}(s, \bar{C}) = A \cdot B \cdot \bar{C} + A \cdot \bar{B} \cdot \bar{C}$$

Assume $\Pr\{A\} = 5.00\text{e} - 6$, $\Pr\{B\} = 5.00\text{e} - 3$ and $\Pr\{C\} = 1.00\text{e} - 2$. This example is interesting for the following reasons:

- Although A is very critical (the failure of A implies the failure of the whole system), its probability of failure is 10 times lower than the probability of the joined failures of components B and C;
- Components B and C play a symmetrical role, but the probability of failure of C is five times the one of B.

The top event probability calculated from BDD is 5.499975×10^{-5} and 5.5×10^{-5} from MCS (in this example, for all indicators, values calculated from MCS are very close to the exact values).

Table 2 presents reliability indicators calculated for system $s = A + B \cdot C$ and the ranking of elements they induce. We put numbers only with a significant decimal digit, but calculations are indeed performed with a much higher precision able to capture even small differences.

Structurally, components B and C play a similar role. As a consequence, they cannot be distinguished really by the DIF (although B is twice as much reliable than C). Their DIF is about 10 times higher than the one of A which reflects the relative weights of failure states in which A is failed and B and C are jointly failed.

In almost all states (namely, all states but $A \cdot B \cdot C$ and $\bar{A} \cdot B \cdot C$), flipping the state of A flips the state of the system. Therefore, MIF(s, A) is very high, although the real contribution of A to the risk is low. Note also that MIF ranks B and C in inverse order of their probabilities.

Finally, RRW(s, A) is higher than RRW(s, B) and RRW(s, C). This is expected for improving the reliability of A and has a greater impact of system reliability than improving the reliability of B and C. Note, however, that RRW does not make it possible to distinguish B and C.

This example shows that there is no such thing as "the" good importance measure. All of them can and probably should be used, but with much care. We shall look at the next section the independently of calculation issues.

Table 2. Reliability indicators for the system $s = A + B \cdot C$.

	A	B	C	Ranking
$\Pr\{E\}$	5.00e − 6	5.00e − 3	1.00e − 2	$A < B < C$
$\mathrm{MIF}(s, E)$	10.00e − 1	10.00e − 3	5.00e − 3	$A > B > C$
$\Pr\{s\|E\}$	1.00	1.00e − 2	5.00e − 3	$A > B > C$
$\Pr\{s\|\bar{E}\}$	5.00e − 5	5.00e − 6	5.00e − 6	$A > B = C$
$\mathrm{crit}(s, E)$	5.00e − 6	5.00e − 5	5.00e − 5	$B = C > A$
$\Pr(s \cdot E)$	5.00e − 6	5.00e − 5	5.00e − 5	$C \approx B > A$
$\Pr(s \cdot \bar{E})$	5.00e − 5	4.98e − 6	4.95e − 6	$A > B \approx C$
$\mathrm{MIF}(s, E)/\Pr\{E\}$	1.82e4	1.82e2	9.09e1	$A > B > C$
$\mathrm{RAW}(s, E)$	1.82e4	1.82e2	9.10e1	$A > B > C$
$\mathrm{RRW}(s, E)$	9.09e − 1	9.09e − 2	9.09e − 2	$A > B = C$
$\mathrm{CIF}(s, E)$	9.09e − 2	9.09e − 1	9.09e − 1	$B = C > A$
$\mathrm{DIF}(s, E)$	9.09e − 2	9.10e − 1	9.10e − 1	$C \approx B > A$
$\overline{\mathrm{crit}}(s, E)$	9.09e − 1	9.05e − 2	9.00e − 2	$A > B \approx C$

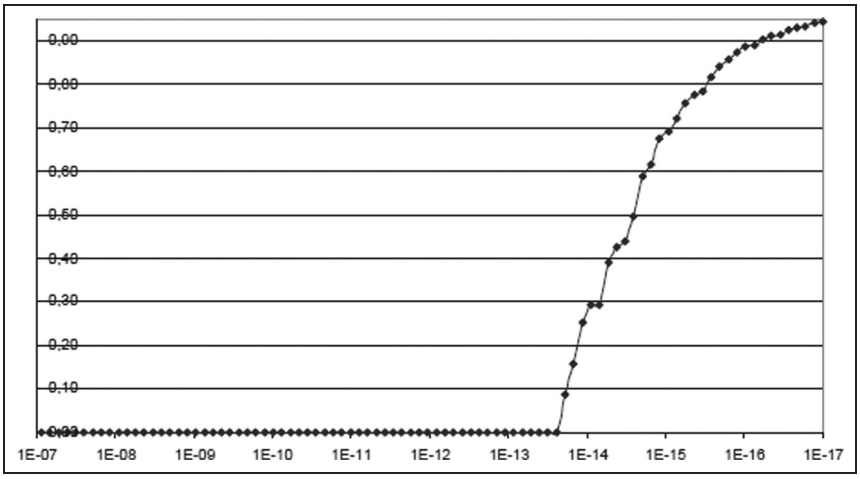

Figure 7. Importance of one of the basic events of a French reference PSA depending on the chosen cutoff.

Calculation issues

For large models (e.g. nuclear PSA), it is not possible to extract all the MCS (and even less to compute the BDD). Cutoffs are therefore applied to "select" only the most probable MCS. Such truncations are safe most of the time for what concerns the calculation of the top event probability, that is, to estimate the global risk.[24] However, importance factors can show a chaotic behavior as the cutoff varies. These phenomena have been first observed by Epstein and Rauzy on American and Japanese PSA[25] and then confirmed by Duflot et al.[26,27] on a reference French PSA. Figure 7 illustrates the effects of truncation threshold on the importance of one of the basic events of a French reference PSA. For a cutoff over 10^{-13}, the basic event does not show up in the MCS, so its risk significance is 0. As the cutoff value decreases, its importance grows dramatically.

To reach very low cutoff values, one has to break the problem into pieces. The Shannon decomposition can be used to do so (this is basically what is proposed in Duflot et al.[27]). Note that this technique has also been proposed to calculate BDD by pieces[28] and that Rauzy proposed recently an efficient algorithm to extract MCS based on the Shannon decomposition.[21]

Conclusion

In this article, we review mathematical and algorithmic foundations of importance factors of coherent systems. We showed that each importance factor characterizes the probability of a set of minterms. This algebraic interpretation clarifies greatly their physical interpretation. It makes it possible also to discard unsuitable indicators such as the so-called Fussel–Vesely importance factor.

Extensions to complex components and groups of components,[29,30] to noncoherent systems,[31] to time-dependent systems[32,33] or to multi-state systems[34] are

out of scope of this article. But we believe that the framework we propose here is a good starting point to analyze them.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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