Handling Epistemic Uncertainty in Fault Trees: New proposal Based on Evidence Theory and Kleene Ternary Decision Diagrams

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***Abstract*— Fault tree (FT) is the most used approach in reliability and safety studies. In most cases, the quantification of the FT top event is carried out either (*i*) without considering uncertainties associated with the basic events probability distribution parameters (assuming single-valued parameters) or (*ii*) using Monte Carlo analysis (MC) to account for that uncertainties (using probability density function (*pdf*)). However, MC approach is not suitable to characterize parameter uncertainties (epistemic uncertainty) for the case where the available data are poor. For that case, interval-valued information (supplied by experts) related to the considered parameters is more appropriate than the MC approach. Within this framework, the present paper propose a new approach addressing epistemic uncertainty in FT based on coupling Dempster-Shafer Theory (DST, also known as Evidence Theory) and the Kleene Ternary Decision Diagrams (Kleene-TDDs). Indeed, DST is used to characterize epistemic uncertainty at basic events level, whereas Kleene-TDDs make it possible to propagate that uncertainty through the fault tree gates up to the top event.**

Keywords-Fault tree; epistemic uncertainty; evidence theory; Kleene-TDDs; Belief (Bel); Plausibility (Pl).

I. Introduction

Fault tree (FT) [1] is the most used and recommended approach in reliability and safety studies. It displays the logical interrelationships of basic events that lead to a predefined undesired event (top event) [1, 2]. The top event quantification requires the prior determination of the parameters characterizing the related basic events probability models (failure rates, repair rates, etc.). These parameters are in general uncertain due to the lack of knowledge regarding the associated stochastic process. This kind of uncertainty is called parameter uncertainty, also known as epistemic uncertainty or subjective uncertainty [3-6]. However, the top event quantification is in most cases carried out without considering that uncertainty by assuming single-valued parameters. This non-consideration of uncertainty may be inappropriate and could therefore lead to erroneous results.

In order to increase the results confidence, uncertainties should be accurately addressed. Among the existing approaches for uncertainty treatment, Monte Carlo analysis (MC) is the most commonly used one, where data uncertainty processing is based on a sampling performed according to probability density functions (*pdfs*) [7]. Still, this classical probabilistic technique is not suitable and can be misleading for the case where the available data are poor [8]. For that case, interval-valued information (vague or subjective information supplied by experts) related to the considered parameters is more appropriate than the MC sampling which requires assumptions about the different *pdf*s to be used. Therefore, a more suitable mathematical framework than the classical probabilistic one is required. For this end, several alternative approaches have been used to handle epistemic uncertainty under data shortcoming [9, 10], namely: interval arithmetic, fuzzy sets [11] and Dempster-shafer theory (DST), also known as evidence theory [12]. In this paper the DST is used to address parameter uncertainty in FT quantification. A comprehensive survey on the application of this approach to the reliability field in general and to the FT in particular can be found in [8, 13].

The present paper propose a new approach addressing uncertainty in FT based on coupling DST and Kleene Ternary Decision Diagrams (Kleene-TDDs). Indeed, DST is used to characterize epistemic uncertainty at basic events level, whereas Kleene-TDDs make it possible to propagate that uncertainty through the FT gates up to the top event. It is worth noticing that Kleene-TDDs provide a natural and perfect framework for uncertainty propagation using DST due to its usefulness for logic representation in the presence of unknown inputs. Furthermore, the encoding of FT via a Kleene-TDD allows reducing the computational complexity encountered when applying adapted version of traditional techniques based on the determination of the minimal cut sets (MCS) or prime implicants, i.e. inclusion-exclusion and sum of disjoint products (SDPs).

The remainder of this paper is organized as follows. Section 2 presents the key concepts related to DST relying on single component reliability. That presentation is limited to what is required regarding the scope of the paper. Section 3 describes briefly BDDs and Kleene-TDDs techniques. Section 4 is focused on FT top event probability derivation from the associated Kleene-TDD. Finally, Section 5 offers a summary of the present work.

II. Dempster-Shafer Theory Basics

Dempster-Shafer theory (DST), also known as evidence theory or belief functions theory, was initiated by Arthur P. Dempster in 1967 [14] and completed later by Glenn Shafer in 1976 [12] to overcome many drawbacks in the traditional Bayesian probability theory. Therefore, DST can be interpreted as a generalization of the Bayesian theory [8]. It is based on three basic measures, namely: the *basic probability assignment* (*bpa* or *m*), the *belief* measure (*Bel*), and the *plausibility* measure (*Pl*).

The definition of these measures is based on the so called *frame of discernment* Ω which represents the definition domain of a given variable. It consists of all mutually exclusive and exhaustive elementary propositions (for instance, all possible states of a system).

$$\begin{cases} \Omega = \{H_1, H_2, \dots, H_n\} \\ H_i \cap H_j = \emptyset, \ \forall i, j = 1, \dots, n. \end{cases} \tag{1}$$

2^Ω is the related power set that comprises all the possible subsets of Ω, including the empty set $\emptyset$.

$$2^\Omega = \{A \mid A \subseteq \Omega\} \tag{2}$$

A. Basic Probability Assignment (bpa)

A basic probability assignment (*bpa*), also called basic belief assignment (*bba*) or belief mass (*m*), is assigned to each subset *A* of the power set 2^Ω. Formally:

$$m: 2^\Omega \rightarrow [0, 1] \tag{3}$$

which satisfies: $\begin{cases} m(\emptyset) = 0 \\ \sum_{A \in 2^\Omega} m(A) = 1 \end{cases}$ (4)

m(*A*) represents the amount of knowledge (available evidence) that supports the subset *A*. Note that the probability distribution is defined on Ω and *bpa* on the power set 2^Ω.

B. Belief (Bel) and Plausibility (Pl) functions

A belief function *Bel* on Ω is a function $Bel : 2^\Omega \rightarrow [0, 1]$ defined as follows:

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{5}$$

Bel(*A*) is obtained by summing the *bpas* of the propositions that totally agree with *A* (the proper subsets of the element A). The inverse formula called the Möbius transform of *Bel* is defined hereafter:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A|-|B|} Bel(B) \tag{6}$$

A plausibility function *Pl* on Ω is a function $Pl : 2^\Omega \rightarrow [0, 1]$ defined as follows:

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) \tag{7}$$

Pl (*A*) is obtained by summing *bpas* of propositions that agree totally or partially with *A* (the subsets that intersect with the element *A*).

Bel (*A*) and *Pl*(*A*) are linked by :

$$\begin{cases} Pl(A) + Bel(\bar{A}) = 1 \\ Pl(\bar{A}) + Bel(A) = 1 \end{cases} \tag{8}$$

where $\bar{A} = \Omega - A$. *Bel*(*A*) and *Pl*(*A*) can be seen as the lower and the upper bounds respectively of the interval that includes the true probability of *A*: $Bel(A) \leq p(A) \leq Pl(A)$.

C. Single component reliability based on DST

Let us consider a binary state component *i*. If x_i denotes the associated basic event, the related frame of discernment $\Omega_i = \{\bar{x}_i, x_i\}$. The corresponding power set $2^{\Omega_i} = \{\emptyset, \{\bar{x}_i\}, \{x_i\}, \{\bar{x}_i, x_i\}\}$. The associated *bpas* at a given time *t* are noted as follows (*t* is omitted on purpose):

$$\begin{cases} m(\{\bar{x}_i\})(t) = m(\bar{x}_i) \\ m(\{x_i\})(t) = m(x_i) \\ m(\{\bar{x}_i, x_i\})(t) = m(\bar{x}_i, x_i) = 1 - m(\bar{x}_i) - m(x_i) \end{cases} \tag{9}$$

The belief and plausibility for the component working and failed sates are obtained by using (5) and (7):

$$\begin{cases} Bel(\{\bar{x}_i\})(t) = Bel(\bar{x}_i) = m(\bar{x}_i) \\ Pl(\{\bar{x}_i\})(t) = Pl(\bar{x}_i) = m(\bar{x}_i) + m(\bar{x}_i, x_i) \\ Bel(\{x_i\})(t) = Bel(x_i) = m(x_i) \\ Pl(\{x_i\})(t) = Pl(x_i) = m(x_i) + m(\bar{x}_i, x_i) \end{cases} \tag{10}$$

For example, if we consider a non-repairable component, the associated probability model is: $1 - e^{-\lambda t}$. If $\lambda \in [\lambda_{min}, \lambda_{max}] = [\underline{\lambda}, \overline{\lambda}]$, then:

$$\begin{cases} Bel(\bar{x}_i) = e^{-\overline{\lambda} t} \ ; \ Pl(\bar{x}_i) = e^{-\underline{\lambda} t} \\ Bel(x_i) = 1 - e^{-\underline{\lambda} t} \ ; \ Pl(x_i) = 1 - e^{-\overline{\lambda} t} \end{cases} \tag{11}$$

The different *bpas* $m(\bar{x}_i)$, $m(x_i)$ and $m(\bar{x}_i, x_i)$ can be then obtained via (10).

III. Kleene Ternary Decision Diagrams

Before introducing Kleene-TDDs, we will first provide a brief overview on Binary Decision Diagrams (BDDs).

A. BDDs

BDDs were originally introduced by Lee [15], and later by Akers [16]. In 1986, Bryant [17] introduced the reduced ordered BDD (ROBDD), allowing efficient representation and manipulation of Boolean functions. Commonly, a BDD is understood to mean the ROBDD. In system reliability, BDDs offer a fast, efficient and accurate way for analyzing coherent and non-coherent fault trees [18].

Let $B = \{0, 1\}$ and let $X = \{x_1, x_2, \dots, x_n\}$ be a set of Boolean variables. The mapping $f : B^n \rightarrow B$ is a Boolean function over the set *X*. The BDD representation is primarily based on Shannon's decomposition (expansion) of *f*:

$$f = x_i \cdot f_{|x_i=1} + \bar{x}_i \cdot f_{|x_i=0} = x_i \cdot f_1 + \bar{x}_i \cdot f_0 \tag{12}$$

where f_1 and f_0 are Boolean functions obtained by evaluating f at $x_i = 1$ and $x_i = 0$, respectively. In terms of the ternary If-Then-Else (ITE) connective [19], (12) can be rewritten as:

$$f = x_i \cdot f_1 + \bar{x}_i \cdot f_0 = ite\ (x_i, f_1, f_0) \tag{13}$$

By choosing a total order over the variables and applying recursively the Shannon decomposition, any Boolean function can be graphically represented as a binary decision tree (for instance, see Fig. 2 for $f = (x_1 \cdot x_2) + x_3$).

The nodes of the binary tree are either without outgoing edges called terminal (sink nodes, leaves) or non-terminal (non-sink or internal nodes). The terminal nodes are labeled with either 0 or 1 (i.e. representing the system being in an operational or a failed state, respectively). Each non-terminal node is labeled by a Boolean variable x_i and has two outgoing edges: 1-edge (then-edge or high-edge) and 0-edge (else-edge or low-edge). Non-terminal nodes encode Boolean functions in the ITE format. Note that a path in the BDD from the root node to a terminal node represents an assignment of values to the variables. The value of the leaf node is the function value for that assignment.

The binary tree is a very space consuming representation. Fortunately, it is possible to compact it by means of the following two reduction rules, which lead to the corresponding BDD. Both reduction rules are sketched in Fig. 1 [20].

- *Merging rule*: this rule is applicable if there are nodes *v* and *w* with the same label, the same 0-successor, and the same 1-successor. We can redirect all edges leading to *v* to the node *w* and we can then delete *v*.
- *Deletion rule*: deleting nodes *v* for which both outgoing edges lead to the same node *w*. It is obvious that we can redirect all edges leading to *v* to the node *w* and that we can delete *v* afterwards.

The resulting BDD is therefore a directed acyclic graph, also called Reduced Ordered BDD (ROBDD). It gives unique form (canonical) to a Boolean function when the order of the input variables is fixed [17].

Logical operations can be performed directly on BDD based on the ITE connective. In this way, the Shannon tree is never built then shrunk [19]. Thus, the BDD encoding a fault tree is obtained by composing the BDDs of its sub-trees. Starting at the bottom of the tree, a BDD is constructed for each basic event (a single-node BDD: *ite* (x_i, 1, 0)) and then combined according to the logic defined by the gate. The BDDs related to the gates are then combined until the top event gate has been reached.

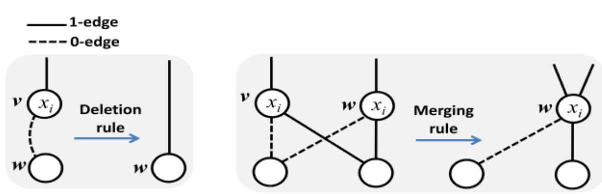

Figure 1. The deletion and merging rules.

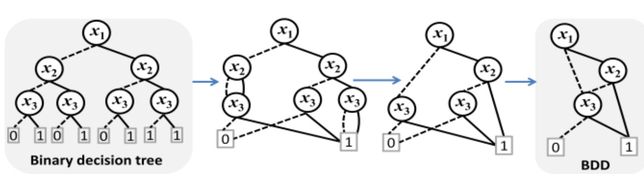

Figure 2. From the binary tree to the BDD of $f = (x_1 \cdot x_2) + x_3$.

The Shannon decomposition applies to probabilities as well. The corresponding algorithm, which is linear in the size of the BDD, is defined by the following recursive equations [21]:

$$\begin{cases} p(0) = 0 \\ p(1) = 1 \\ p(f) = p(x_i) \cdot p(f_1) + \big(1 - p(x_i)\big) \cdot p(f_0) \end{cases} \tag{14}$$

where $p(x_i)$ stands for the failure probability related to the basic event x_i. When x_i is the root node of the entire BDD, $p(f)$ gives the top event probability.

B. Kleene-TDDs

TDDs are similar to BDDs, except that each non-terminal node has three successors (children). A survey on TDDs is addressed in [22]. With connection to system reliability, TDDs are mainly used for studying phased-mission systems [23, 24] and non-coherent fault trees [18]. In this paper, only a particular kind of TDDs is presented and used for uncertainty propagation namely: Kleene-TDDs. They are the TDDs introduced by Jennings [25] and called as such in [26].

Kleene-TDDs represent the Kleene function [27] $\mathcal{F}: T^{\mathrm{n}} \to T$, $T = \{0, 1, u\}$ of a two-valued logic function $f: B^{\mathrm{n}} \to B$, where *u* is the truth value showing an unknown input. Let $Y = \{y_1, y_2, \dots, y_n\}$ be a ternary vector, where $y_i \in T$. $A(Y)$ denotes the set of all the binary vectors that are obtained by replacing all *u* with 0 or 1.

$$\mathcal{F}(Y) = \begin{cases} 0 \;\; if \; f(A(Y)) = \{0\} \\ 1 \;\; if \; f(A(Y)) = \{1\} \\ u \;\; if \; f(A(Y)) = \{0, 1\} \end{cases} \tag{15}$$

where $f(A(Y)) = \{f(X) |\, X \in A(Y)\}$

In other words, if all the vectors in $A(Y)$ are mapped to 0, then $\mathcal{F}(Y) = 0$; if all the vectors are mapped to 1, then $\mathcal{F}(Y) = 1$; and if some vectors are mapped to 0 and others are mapped to 1, then $\mathcal{F}(Y) = u$. Note that f uniquely defines $\mathcal{F}$.

A Kleene-TDD is easy to construct as shown in Fig. 3. [22]. The rightmost sub-graph represents the alignment of f_0 and f_1. The alignment is a ternary operator defined by Kleene:

$$x \odot y = \begin{cases} x \;\; if \; x = y \\ u \;\; otherwise \end{cases} = ite(x = y, x, u) \tag{16}$$

where $x, y \in T$.

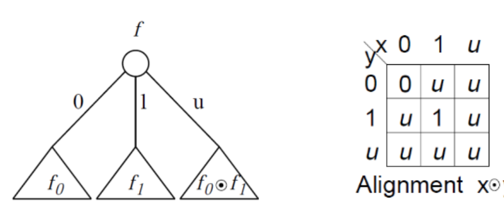

Figure 3. Kleene-TDDs principle.

Fig. 4 gives an example of constructing the Kleene-TDD related to the expression $f = (x_1 \cdot x_2) + x_3$ starting from its binary tree given in Fig. 2. Note that we can directly use the corresponding BDD for deriving the Kleene-TDD. The topmost u-edge is generated and the sub-graph is created based on the two sub-trees related to the 0-edge (f_0) and 1-edge (f_1) using the alignment operation. In the resulting sub-tree a terminal node labeled with 'u' appears wherever the two corresponding original terminals do not match each other. The process is then repeated recursively down the entire tree. The u-successor (child) at any non-terminal node represents the alignment of the 0 and 1-successors for the same non-terminal node. By eliminating all the redundant nodes (deletion rule) and sharing all the equivalent sub-graphs (merging rule), the resulting ternary decision tree is reduced to a directed acyclic graph: reduced ordered Kleene-TDD (Kleene-TDD for short). Note that if we remove all the u-edges from the Kleene-TDD, we obtain the corresponding reduced ordered BDD. One may therefore understand that the Kleene-TDD is canonical for a given variable ordering.

Kleene-TDDs are directed acyclic graph containing terminal and non-terminal nodes. Each non-terminal node is labeled with a variable (x_i) and having three outgoing edges (0-edge, 1-edge and u-edge). A terminal nodes is labeled with a value $\in \{0, 1, u\}$ that represents $\mathcal{F}(Y_i)$, Y_i being the variables assignment from the root node to the corresponding terminal node.

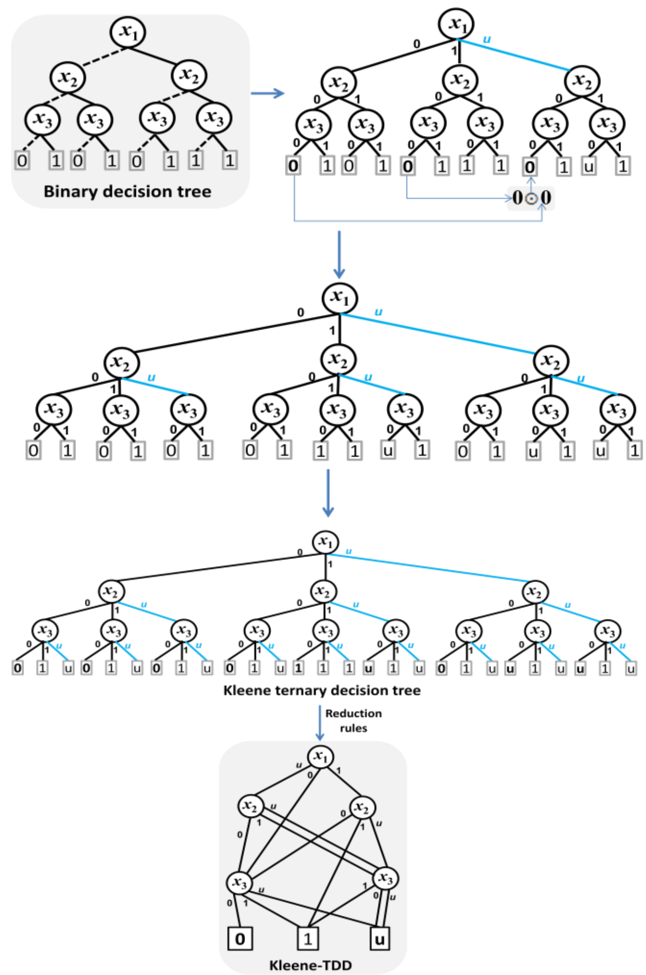

Figure 4. Kleene-TDD construction for $f = (x_1 \cdot x_2) + x_3$.

IV. Top Event (TE) Probability Quntification

For a given top event (TE), we consider the frame of discernment $\Omega_{\mathrm{TE}} = \{\{\overline{\mathrm{TE}}\}, \{\mathrm{TE}\}\}$. The corresponding power set is then: $2^{\Omega_{\mathrm{TE}}} = \{\emptyset, \{\overline{\mathrm{TE}}\}, \{\mathrm{TE}\}, \{\overline{\mathrm{TE}}, \mathrm{TE}\}\}$. We describe in the following how to compute the probability of TE in terms of belief (*Bel*(TE)) and plausibility (*Pl*(TE)). These two measures characterize the interval in which lies the true TE probability: $p_{TE} \in [Bel(\mathrm{TE}),\ Pl(\mathrm{TE})]$. We recall that the associated basic event probabilities are uncertain: $p(x_i) \in [Bel(x_i),\ Pl(x_i)]$.

Once the Kleene-TDD is constructed, a similar approach to the BDDs one can be used to derive the TE probability measures, i.e. *Bel*(TE) and *Pl*(TE). It is worth noticing that a path (from the root) having a terminal node labeled with 1 (resp. 0) represents a given assignment of variables leading to the TE (resp. $\overline{\mathrm{TE}}$). Moreover, if a terminal node for a path is labeled with u, then that path leads to an unknown state of the system, i.e. the set $\{\overline{\mathrm{TE}}, \mathrm{TE}\}$ in $2^{\Omega_{\mathrm{TE}}}$. Note that all the paths of the Kleene-TDD are disjoint. For the quantification process, the quantities associated with each 0-edge, 1-edge and u-edge leaving a non-terminal node labeled with the variable (basic event) x_i are the *bpas* $m(\bar{x}_i) = Bel(\bar{x}_i) = 1 - Pl(x_i)$, $m(x_i) = Bel(x_i)$ and $m(\bar{x}_i,\ x_i) = 1 - [Bel(\bar{x}_i) + Bel(x_i)] = Pl(x_i) - Bel(x_i)$, respectively (Fig. 5). The *bpas* are derived using (9) and (10).

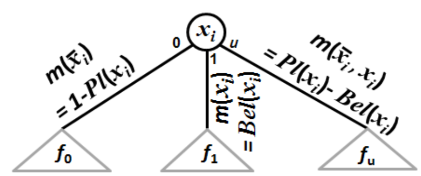

Figure 5. A Kleene-TDD node with *bpa* information

According to the above statements, the fault tree *TE* belief, $Bel(TE)$, can thus be simply calculated as the sum of *bpas* of all the paths from the root to terminal node 1, where the *bpa* of a given path is the product of the *bpas* involved in that path. Similarly, the fault tree *TE* plausibility, $Pl(TE)$, can be given by the sum of *bpas* of all the paths from the root to terminal nodes 1or u. This quantification can be performed recursively by simply generalizing the algorithm provided in (15):

$$p(f) = m(\bar{x}_i)\ \cdot p(f_0) + m(x_i) \cdot p(f_1) + m(\bar{x}_i, x_i) \cdot p(f_u)$$

$$= [1 - Pl(x_i)]\ \cdot p(f_0) + Bel(x_i) \cdot p(f_1) + [Pl(x_i) - Bel(x_i)] \cdot p(f_u) \quad (17)$$

where $p(f)$ represents either the belief or the plausibility measures depending on the exit condition of the recursive algorithm. According to what we have already mentioned, this condition can be defined as follows:

$$\begin{cases} p(0) = 0 \\ p(1) = 1 \\ p(u) = \begin{cases} 0 & if\ p(f) = Bel(f) \\ 1 & if\ p(f) = Pl(f) \end{cases} \end{cases} \quad (18)$$

when x_i is the root node of the entire Kleene-TDD, $p(f)$ gives the *TE* belief or plausibility. It should be noted that the exit condition with respect to $p(u)$ can be interpreted as a switching operation: a terminal node labeled with u becomes labeled with 0 for the belief measure, whereas it becomes labeled with 1 for the plausibility measure.

In order to illustrate the algorithm defined by (17) and (18), let us consider again the Boolean function $f = (x_1 \cdot x_2) + x_3$ as a fault tree TE. The corresponding Kleene-TDD is depicted in Fig. 4. Starting from the root node x_1, (17) yields the subsequent relations.

$$p(f) = [1 - Pl(x_1)] \cdot p(f_0) + Bel(x_1) \cdot p(f_1) + [Pl(x_1) - Bel(x_1)] \cdot p(f_u)$$

$$p(f_0) = [1 - Pl(x_3)] \cdot p(0) + Bel(x_3) \cdot p(1) + [Pl(x_3) - Bel(x_3)] \cdot p(u)$$

$$p(f_1) = [1 - Pl(x_2)] \cdot p(f_{10}) + Bel(x_2) \cdot p(1) + [Pl(x_2) - Bel(x_2)] \cdot p(f_{1u})$$

$$p(f_{10}) = p(f_0)$$

$$p(f_{1u}) = [1 - Pl(x_3)] \cdot p(u) + Bel(x_3) \cdot p(1) + [Pl(x_3) - Bel(x_3)] \cdot p(u)$$

$$p(f_u) = [1 - Pl(x_2)] \cdot p(f_{u0}) + Bel(x_2) \cdot p(f_{u1}) + [Pl(x_2) - Bel(x_2)] \cdot p(f_{uu})$$

$$p(f_{u0}) = p(f_0)$$

$$p(f_{u1}) = p(f_{uu}) = p(f_{1u})$$

According to (18), the TE belief and plausibility are computed by setting $p(u)$ equals to 0 and 1, respectively. For numeric application, we assume that the basic events x_1, x_2 and x_3 characterize failed states of non-repairable components. The corresponding uncertain failure rates (h^{-1}) are $\lambda_1 \in [0.002,\ 0.003]$, $\lambda_2 \in [0.0045,\ 0.0055]$ and $\lambda_3 \in [0.0055,\ 0.007]$. $Bel(x_i)$ and $Pl(x_i)$ are derived using (11). The TE instantaneous belief and plausibility are provided in Fig. 6. For instance, if $t = 250\ h$, the TE probability $p(\mathrm{TE}) \in [0.8143,\ 0.8947]$.

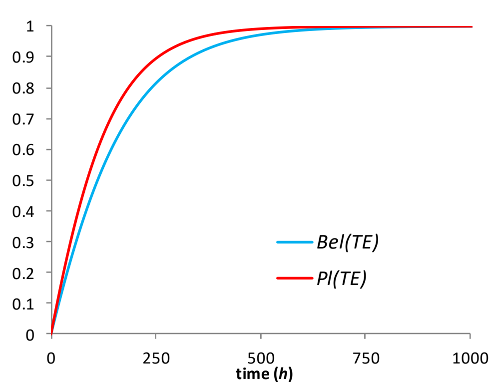

Figure 6. TE belief and plausibility against time.

V. Conclusion

This paper deals with epistemic uncertainty (parametric uncertainty) encountered during FT top event probability assessment. More precisely, we considered the case where the FT basic events uncertainty, due to poor availability of reliability data, is handled using Dempster-Shafer Theory (DST), i.e. the basic event probability is given by an interval, where its lower and upper bounds are referred to as belief (*Bel*) and plausibility (*Pl*), respectively. We note that this approach is considered as one of the primary mathematical framework for knowledge representation under uncertainty. In order to propagate the uncertainty related to the different basic events input parameters to the FT top event probability, we proposed a new approach based on a specific kind of ternary decision diagrams (TDDs), namely Kleene-TDDs. We notice that Kleene-TDDs offer a perfect framework for uncertainty handling when basic events uncertainties are characterized using DST, for both coherent and non-coherent fault trees. A generalization of the BDD recursive algorithm for the top event probability is given so as to compute the top event belief (*Bel*(TE)) and plausibility (*Pl*(TE)).

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